Extension

Graph Quadratic Functions in Vertex Form

GOAL Graph quadratic functions in vertex form.

Key Vocabulary
- **vertex form**

You have graphed quadratic functions in standard form. Quadratic functions can also be written in **vertex form**, $y = a(x - h)^2 + k$ where $a \neq 0$. In this form, the vertex of the graph can be easily determined.

COMMON CORE

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

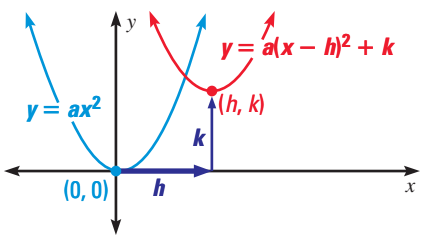
KEY CONCEPT *For Your Notebook*

Graph of Vertex Form $y = a(x - h)^2 + k$

The graph of $y = a(x - h)^2 + k$ is the graph of $y = ax^2$ translated h units horizontally and k units vertically.

Characteristics of the graph of $y = a(x - h)^2 + k$:

- The vertex is (h, k).
- The axis of symmetry is $x = h$.
- The graph opens up if $a > 0$, and the graph opens down if $a < 0$.



EXAMPLE 1 Graph a quadratic function in vertex form

Graph $y = -(x + 2)^2 + 3$.

Solution

STEP 1 **Identify** the values of a, h, and k: $a = -1$, $h = -2$, and $k = 3$. Because $a < 0$, the parabola opens down.

STEP 2 **Draw** the axis of symmetry, $x = -2$.

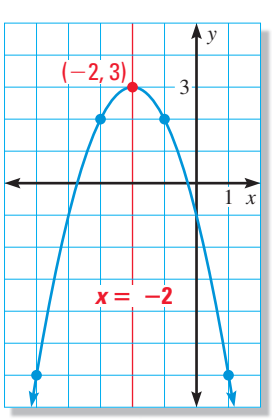
STEP 3 **Plot** the vertex $(h, k) = (-2, 3)$.

STEP 4 **Plot** four points. Evaluate the function for two x-values less than the x-coordinate of the vertex.

$x = -3$: $y = -(-3 + 2)^2 + 3 = 2$

$x = -5$: $y = -(-5 + 2)^2 + 3 = -6$

Plot the points $(-3, 2)$ and $(-5, -6)$ and their reflections, $(-1, 2)$ and $(1, -6)$, in the axis of symmetry.



STEP 5 **Draw** a parabola through the plotted points.

EXAMPLE 2 Graph a quadratic function

Graph $y = x^2 - 8x + 11$.

Solution

STEP 1 **Write** the function in vertex form by completing the square.

$y = x^2 - 8x + 11$ — Write original function.

$y + \square = (x^2 - 8x + \square) + 11$ — Prepare to complete the square.

$y + 16 = (x^2 - 8x + 16) + 11$ — Add $\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$ to each side.

$y + 16 = (x - 4)^2 + 11$ — Write $x^2 - 8x + 16$ as a square of a binomial.

$y = (x - 4)^2 - 5$ — Subtract 16 from each side.

STEP 2 **Identify** the values of a, h, and k: $a = 1$, $h = 4$, and $k = -5$. Because $a > 0$, the parabola opens up.

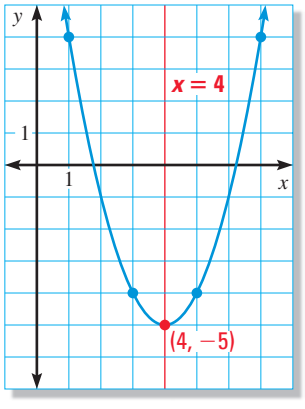
STEP 3 **Draw** the axis of symmetry, $x = 4$.

STEP 4 **Plot** the vertex $(h, k) = (4, -5)$.

STEP 5 **Plot** four more points. Evaluate the function for two x-values less than the x-coordinate of the vertex.

$\boldsymbol{x = 3}$: $y = (3 - 4)^2 - 5 = -4$
$\boldsymbol{x = 1}$: $y = (1 - 4)^2 - 5 = 4$

Plot the points $(3, -4)$ and $(1, 4)$ and their reflections, $(5, -4)$ and $(7, 4)$, in the axis of symmetry.



STEP 6 **Draw** a parabola through the plotted points.

PRACTICE

EXAMPLE 1 for Exs. 1–6

Graph the quadratic function. Label the vertex and axis of symmetry.

1. $y = (x + 2)^2 - 5$ **2.** $y = -(x - 4)^2 + 1$ **3.** $y = x^2 + 3$

4. $y = 3(x - 1)^2 - 2$ **5.** $y = -2(x + 5)^2 - 2$ **6.** $y = -\frac{1}{2}(x + 4)^2 + 4$

EXAMPLE 2 for Exs. 7–12

Write the function in vertex form, then graph the function. Label the vertex and axis of symmetry.

7. $y = x^2 - 12x + 36$ **8.** $y = x^2 + 8x + 15$ **9.** $y = -x^2 + 10x - 21$

10. $y = 2x^2 - 12x + 19$ **11.** $y = -3x^2 - 6x - 1$ **12.** $y = -\frac{1}{2}x^2 - 6x - 21$

13. Write an equation in vertex form of the parabola shown. Use the coordinates of the vertex and the coordinates of a point on the graph to write the equation.