9.6 Solve Quadratic Equations by the Quadratic Formula

Before You solved quadratic equations by completing the square.

Now You will solve quadratic equations using the quadratic formula.

Why? So you can solve a problem about film production, as in Example 3.

Key Vocabulary
- **quadratic formula**

By completing the square for the quadratic equation $ax^2 + bx + c = 0$, you can develop a formula that gives the solutions of any quadratic equation in standard form. This formula is called the **quadratic formula**.

KEY CONCEPT *For Your Notebook*

The Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a \neq 0 \text{ and } b^2 - 4ac \geq 0.$$

EXAMPLE 1 Standardized Test Practice

What are the solutions of $3x^2 + 5x = 8$?

Ⓐ -1 and $-\frac{8}{3}$ Ⓑ -1 and $\frac{8}{3}$ Ⓒ 1 and $-\frac{8}{3}$ Ⓓ 1 and $\frac{8}{3}$

ANOTHER WAY
Instead of solving the equation, you can check the answer choices in the equation.

Solution

$3x^2 + 5x = 8$ — Write original equation.

$3x^2 + 5x - 8 = 0$ — Write in standard form.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic formula

$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-8)}}{2(3)}$ — Substitute values in the quadratic formula: $a = 3$, $b = 5$, and $c = -8$.

$= \frac{-5 \pm \sqrt{121}}{6}$ — Simplify.

$= \frac{-5 \pm 11}{6}$ — Simplify the square root.

The solutions of the equation are $\frac{-5 + 11}{6} = 1$ and $\frac{-5 - 11}{6} = -\frac{8}{3}$.

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 2 Solve a quadratic equation

Solve $2x^2 - 7 = x$.

$2x^2 - 7 = x$ — Write original equation.

$2x^2 - x - 7 = 0$ — Write in standard form.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic formula

$= \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-7)}}{2(2)}$ — Substitute values in the quadratic formula: $a = 2$, $b = -1$, and $c = -7$.

$= \dfrac{1 \pm \sqrt{57}}{4}$ — Simplify.

▶ The solutions are $\dfrac{1 + \sqrt{57}}{4} \approx 2.14$ and $\dfrac{1 - \sqrt{57}}{4} \approx -1.64$.

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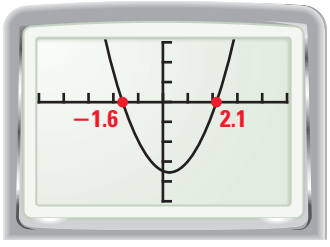
CHECK Write the equation in standard form, $2x^2 - x - 7 = 0$. Then graph the related function $y = 2x^2 - x - 7$. The x-intercepts are about -1.6 and 2.1. So, each solution checks.



GUIDED PRACTICE for Examples 1 and 2

Use the quadratic formula to solve the equation. Round your solutions to the nearest hundredth, if necessary.

1. $x^2 - 8x + 16 = 0$ **2.** $3n^2 - 5n = -1$ **3.** $4z^2 = 7z + 2$

EXAMPLE 3 Use the quadratic formula

FILM PRODUCTION For the period 1971–2001, the number y of films produced in the world can be modeled by the function $y = 10x^2 - 94x + 3900$ where x is the number of years since 1971. In what year were 4200 films produced?

Solution

$y = 10x^2 - 94x + 3900$ — Write function.

$4200 = 10x^2 - 94x + 3900$ — Substitute 4200 for y.

$0 = 10x^2 - 94x - 300$ — Write in standard form.

$x = \dfrac{-(-94) \pm \sqrt{(-94)^2 - 4(10)(-300)}}{2(10)}$ — Substitute values in the quadratic formula: $a = 10$, $b = -94$, and $c = -300$.

$= \dfrac{94 \pm \sqrt{20{,}836}}{20}$ — Simplify.

The solutions of the equation are $\dfrac{94 + \sqrt{20{,}836}}{20} \approx 12$ and $\dfrac{94 - \sqrt{20{,}836}}{20} \approx -3$.

▶ There were 4200 films produced about 12 years after 1971, or in 1983.

INTERPRET SOLUTIONS
The solution -3 can be ignored because -3 represents the year 1968, which is not in the given time period.

GUIDED PRACTICE for Example 3

4. WHAT IF? In Example 3, find the year when 4750 films were produced.

CONCEPT SUMMARY *For Your Notebook*

Methods for Solving Quadratic Equations

Method	When to Use
Factoring	Use when a quadratic equation can be factored easily.
Graphing	Use when approximate solutions are adequate.
Finding square roots	Use when solving an equation that can be written in the form $x^2 = d$.
Completing the square	Can be used for *any* quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and b is an even number.
Quadratic formula	Can be used for *any* quadratic equation.

EXAMPLE 4 Choose a solution method

Tell what method you would use to solve the quadratic equation. *Explain* your choice(s).

a. $10x^2 - 7 = 0$ **b.** $x^2 + 4x = 0$ **c.** $5x^2 + 9x - 4 = 0$

Solution

a. The quadratic equation can be solved using square roots because the equation can be written in the form $x^2 = d$.

b. The equation can be solved by factoring because the expression $x^2 + 4x$ can be factored easily. Also, the equation can be solved by completing the square because the equation is of the form $ax^2 + bx + c = 0$ where $a = 1$ and b is an even number.

c. The quadratic equation cannot be factored easily, and completing the square will result in many fractions. So, the equation can be solved using the quadratic formula.

GUIDED PRACTICE for Example 4

Tell what method you would use to solve the quadratic equation. *Explain* your choice(s).

5. $x^2 + x - 6 = 0$ **6.** $x^2 - 9 = 0$ **7.** $x^2 + 6x = 5$

9.6 EXERCISES

HOMEWORK KEY

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 19 and 47
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 25, and 50
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 49

SKILL PRACTICE

1. **VOCABULARY** What formula can be used to solve any quadratic equation?

2. ★ **WRITING** What method(s) would you use to solve $-x^2 + 8x = 1$? *Explain* your choice(s).

EXAMPLES 1 and 2 for Exs. 3–27

SOLVING QUADRATIC EQUATIONS **Use the quadratic formula to find the roots of the equation. Round your solutions to the nearest hundredth, if necessary.**

3. $x^2 + 5x - 104 = 0$
4. $4x^2 - x - 18 = 0$
5. $6x^2 - 2x - 28 = 0$
6. $m^2 + 3m + 1 = 0$
7. $-z^2 + z + 14 = 0$
8. $-2n^2 - 5n + 16 = 0$
9. $4w^2 + 20w + 25 = 0$
10. $2t^2 + 3t - 11 = 0$
11. $-6g^2 + 9g + 8 = 0$

12. ★ **MULTIPLE CHOICE** What are the solutions of $10x^2 - 3x - 1 = 0$?

Ⓐ $-\frac{1}{5}$ and $-\frac{1}{2}$ Ⓑ $-\frac{1}{5}$ and $\frac{1}{2}$ Ⓒ $\frac{1}{5}$ and $-\frac{1}{2}$ Ⓓ $\frac{1}{5}$ and $\frac{1}{2}$

SOLVING QUADRATIC EQUATIONS **Use the quadratic formula to solve the equation. Round your solutions to the nearest hundredth, if necessary.**

13. $x^2 - 5x = 14$
14. $3x^2 - 4 = 11x$
15. $9 = 7x^2 - 2x$
16. $2m^2 + 9m + 7 = 3$
17. $-10 = r^2 - 10r + 12$
18. $3g^2 - 6g - 14 = 3g$
19. $6z^2 = 2z^2 + 7z + 5$
20. $8h^2 + 8 = 6 - 9h$
21. $4t^2 - 3t = 5 - 3t^2$
22. $-4y^2 - 3y + 3 = 2y + 4$
23. $7n + 5 = -3n^2 + 2$
24. $5w^2 + 4 = w + 6$

25. ★ **MULTIPLE CHOICE** What are the solutions of $x^2 + 14x = 2x - 11$?

Ⓐ -2 and -22 Ⓑ -1 and -11 Ⓒ 1 and 11 Ⓓ 2 and 22

ERROR ANALYSIS ***Describe*** **and correct the error in solving the equation.**

26. $7x^2 - 5x - 1 = 0$

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4(7)(-1)}}{2(7)}$$
$$= \frac{-5 \pm \sqrt{53}}{14}$$
$x \approx -0.88$ and $x \approx 0.16$ ✗

27. $-2x^2 + 3x = 1$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(-2)(1)}}{2(-2)}$$
$$= \frac{-3 \pm \sqrt{17}}{-4}$$
$x \approx -0.28$ and $x \approx 1.78$ ✗

EXAMPLE 4 for Exs. 28–33

CHOOSING A METHOD **Tell what method(s) you would use to solve the quadratic equation. *Explain* your choice(s).**

28. $3x^2 - 27 = 0$
29. $5x^2 = 25$
30. $2x^2 - 12x = 0$
31. $m^2 + 5m + 6 = 0$
32. $z^2 - 4z + 1 = 0$
33. $-10g^2 + 13g = 4$

SOLVING QUADRATIC EQUATIONS **Solve the quadratic equation using any method. Round your solutions to the nearest hundredth, if necessary.**

34. $-2x^2 = -32$

35. $x^2 - 8x = -16$

36. $x^2 + 2x - 6 = 0$

37. $x^2 = 12x - 36$

38. $x^2 + 4x = 9$

39. $-4x^2 + x = -17$

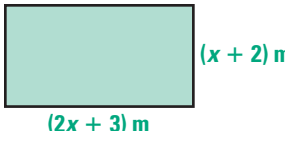
40. $11x^2 - 1 = 6x^2 + 2$

41. $-2x^2 + 5 = 3x^2 - 10x$

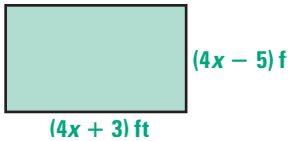
42. $(x + 13)^2 = 25$

GEOMETRY **Use the given area A of the rectangle to find the value of x. Then give the dimensions of the rectangle.**

43. $A = 91$ m^2



44. $A = 209$ ft^2



45. **CHALLENGE** The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$. Find the mean of the solutions. How is the mean of the solutions related to the graph of $y = ax^2 + bx + c$? *Explain.*

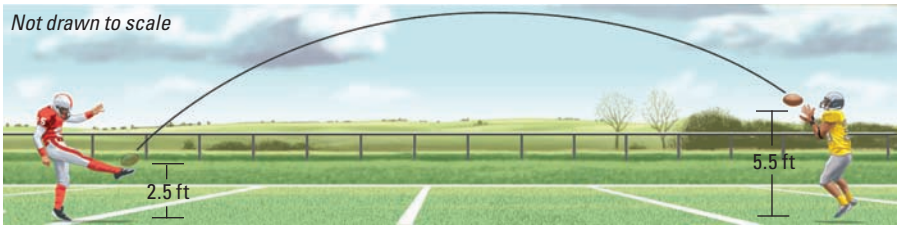
PROBLEM SOLVING

EXAMPLE 3 for Exs. 46–47

46. **ADVERTISING** For the period 1990–2000, the amount of money y (in billions of dollars) spent on advertising in the U.S. can be modeled by the function $y = 0.93x^2 + 2.2x + 130$ where x is the number of years since 1990. In what year was 164 billion dollars spent on advertising?

47. **CELL PHONES** For the period 1985–2001, the number y (in millions) of cell phone service subscribers in the U.S. can be modeled by the function $y = 0.7x^2 - 4.3x + 5.5$ where x is the number of years since 1985. In what year were there 16,000,000 cell phone service subscribers?

48. **MULTI-STEP PROBLEM** A football is punted from a height of 2.5 feet above the ground and with an initial vertical velocity of 45 feet per second.



a. Use the vertical motion model to write an equation that gives the height h (in feet) of the football as a function of the time t (in seconds) after it has been punted.

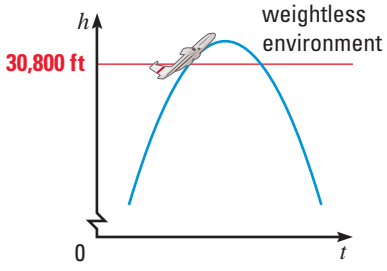
b. The football is caught 5.5 feet above the ground as shown in the diagram. Find the amount of time that the football is in the air.

49. **MULTIPLE REPRESENTATIONS** For the period 1997–2002, the number y (in thousands) of 16- and 17-year-olds employed in the United States can be modeled by the function $y = -46.7x^2 + 169x + 2650$ where x is the number of years since 1997.

a. Solving an Equation Write and solve an equation to find the year during which 2,500,000 16- and 17-year-olds were employed.

b. Drawing a Graph Graph the function on a graphing calculator. Use the *trace* feature to find the year when 2,500,000 16- and 17-year-olds were employed. Use the graph to check your answer from part (a).

50. ★ **SHORT RESPONSE** NASA creates a weightless environment by flying a plane in a series of parabolic paths. The height h (in feet) of a plane after t seconds in a parabolic flight path can be modeled by the graph of $h = -11t^2 + 700t + 21{,}000$. The passengers experience a weightless environment when the height of the plane is greater than or equal to 30,800 feet. Find the period of weightlessness on such a flight. *Explain.*



51. **CHALLENGE** Mineral deposits have formed a uniform coating that is 4 millimeters thick on the inside of a water pipe. The cross-sectional area of the pipe has decreased by 10%. What was the original diameter of the pipe (to the nearest tenth of a millimeter)?

QUIZ

Solve the equation using square roots.

1. $3x^2 - 48 = 0$ **2.** $-6x^2 = -24$ **3.** $x^2 + 5 = 16$

Solve the equation by completing the square.

4. $x^2 + 2x + 6 = 0$ **5.** $x^2 + 10x - 12 = 0$ **6.** $x^2 - 8x = -6$

7. $x^2 - 12x = 30$ **8.** $x^2 - 5x = -\frac{9}{4}$ **9.** $x^2 + x = -7.75$

Solve the equation using the quadratic formula.

10. $x^2 + 4x + 1 = 0$ **11.** $-3x^2 + 3x = -1$ **12.** $4x^2 - 11x = 3$