Extension Derive the Quadratic Formula

GOAL Solve quadratic equations and check solutions.

COMMON CORE

CC.9-12.A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

You have learned how to find solutions of quadratic equations using the quadratic formula. You can use the method of completing the square and the quotient property of radicals to derive the quadratic formula.

$ax^2 + bx + c = 0$	**Write standard form of a quadratic equation.**
$ax^2 + bx = -c$	**Subtract c from each side.**
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	**Divide each side by a, $a \neq 0$.**
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	**Add $\left(\frac{b}{2a}\right)^2$ to each side to complete the square.**
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$	**Write left side as the square of a binomial.**
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	**Simplify right side.**
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	**Take square roots of each side.**
$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$	**Quotient property of radicals**
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**Subtract $\frac{b}{2a}$ from each side.**

SOLVING QUADRATIC EQUATIONS You can use the quadratic formula and properties of radicals to solve quadratic equations.

EXAMPLE 1 Solve an equation

Solve $x^2 - 6x + 3 = 0$.

Solution

$x^2 - 6x + 3 = 0$	**Identify $a = 1$, $b = -6$, and $c = 3$.**
$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)}$	**Substitute values in the quadratic formula.**
$= \frac{6 \pm \sqrt{24}}{2}$	**Simplify.**
$= \frac{6 \pm \sqrt{4 \cdot 6}}{2}$	**Product property of radicals**
$= \frac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}$	**Simplify.**

▶ The solutions of the equation are $3 + \sqrt{6}$ and $3 - \sqrt{6}$.

EXAMPLE 2 Check the solutions of an equation

Check the solutions of the equation from Example 1.

Solution

The solutions of $x^2 - 6x + 3 = 0$ are $3 + \sqrt{6}$ and $3 - \sqrt{6}$. You can check each solution by substituting it into the original equation.

Check $x = 3 + \sqrt{6}$:

$x^2 - 6x + 3 = 0$	Write original equation.
$(3 + \sqrt{6})^2 - 6(3 + \sqrt{6}) + 3 \stackrel{?}{=} 0$	Substitute $3 + \sqrt{6}$ for x.
$9 + 6\sqrt{6} + 6 - 18 - 6\sqrt{6} + 3 \stackrel{?}{=} 0$	Multiply.
$0 = 0$ ✓	Solution checks.

Check $x = 3 - \sqrt{6}$:

$x^2 - 6x + 3 = 0$	Write original equation.
$(3 - \sqrt{6})^2 - 6(3 - \sqrt{6}) + 3 \stackrel{?}{=} 0$	Substitute $3 - \sqrt{6}$ for x.
$9 - 6\sqrt{6} + 6 - 18 + 6\sqrt{6} + 3 \stackrel{?}{=} 0$	Multiply.
$0 = 0$ ✓	Solution checks.

PRACTICE

EXAMPLES 1 and 2 for Exs. 1–18

Solve the equation using the quadratic formula. Check the solution.

1. $x^2 + 4x + 2 = 0$
2. $x^2 + 6x - 1 = 0$
3. $x^2 + 8x + 8 = 0$
4. $x^2 - 7x + 1 = 0$
5. $3x^2 + 6x - 1 = 0$
6. $2x^2 - 4x - 3 = 0$
7. $5x^2 - 2x - 2 = 0$
8. $4x^2 + 10x + 3 = 0$
9. $x^2 - x - 3 = 0$
10. $x^2 - 2x - 8 = 0$
11. $-x^2 + 7x + 3 = 0$
12. $x^2 + 3x - 9 = 0$
13. $-\frac{5}{2}x^2 + 10x - 5 = 0$
14. $\frac{1}{2}x^2 + 3x - 9 = 0$
15. $3x^2 - 2 = 0$
16. $-2x^2 - 7x = 0$
17. $3x^2 + x = 6$
18. $x^2 - 4x = -2$

19. Show that $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$ are solutions of $ax^2 + bx + c = 0$ by substituting.

20. Derive a formula to find solutions of equations that have the form $ax^2 + x + c = 0$. Use your formula to find solutions of $-2x^2 + x + 8 = 0$.

21. Find the sum and product of $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$. Write a quadratic expression whose solutions have a sum of 2 and a product of $\frac{1}{2}$.

22. What values can a have in the equation $ax^2 + 12x + 3 = 0$ in order for the equation to have one or two real solutions? *Explain.*