Name ______________________ Date ____________

CHAPTER 9

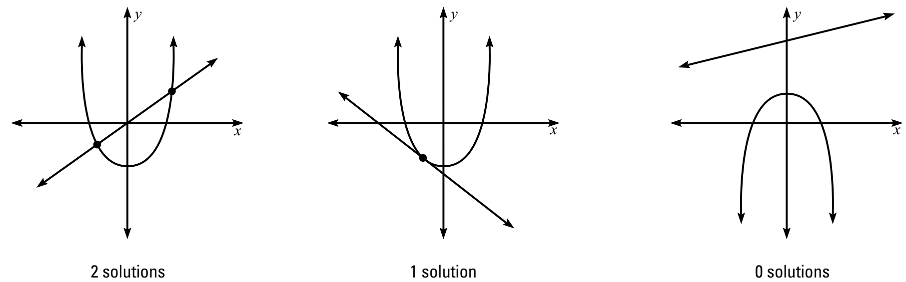
Systems of Equations with at Least One Nonlinear Equation

The solution to a system of equations can be found using algebraic methods or by graphing.

KEY CONCEPT

Solving a System of One Quadratic Equation and One Linear Equation

There are three cases to consider for a system of one quadratic equation and one linear equation.



A system of one quadratic equation and one linear equation gives a graph of one parabola and one line.

The first case is a system with two solutions. In other words, the graphs of the parabola and line intersect in two places. You can solve a system by graphing using a graphing calculator.

EXAMPLE 1

Solve by graphing: two solutions

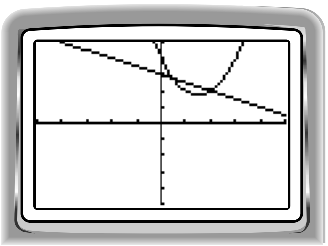
Solve the system by graphing: $x + 2y = 6$
$y = x^2 - 3x + 4$

Solution:

Write each equation as a function of x. Use a graphing calculator to graph the following system.

$$y_1 = -\frac{1}{2}x + 3$$

$$y_2 = x^2 - 3x + 4$$



You can see that there are two intersection points. Since there are two intersection points, there are two solutions. Use the intersection option in the CALC menu of the graphing calculator to find the coordinates of the points of intersection.

The solutions of the system are (0.5, 2.75), (2, 2). ■

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Name ________________________ Date ______________

CHAPTER 9

Systems of Equations with at Least One Nonlinear Equation *continued*

Next, solve a system algebraically.

EXAMPLE 2 Solving algebraically: two solutions

Solve the system: $y = x^2 + 1$
$y = 5$

Solution:

Obtain the following equation by substitution: $x^2 + 1 = 5$

Solve for x: $x^2 = 4$
$x = \pm 2$

Because $y = 5$ for all values of x, the solutions are $(-2, 5)$, $(2, 5)$. You should always check that both ordered pairs are solutions to both equations. ■

The second case is a system of equations with no solution. In other words, the graphs of the parabola and line do not intersect.

EXAMPLE 3 Solve by graphing: no solution

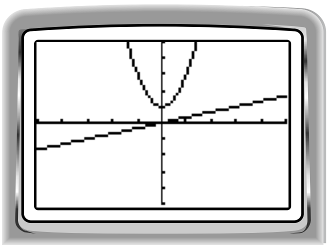
Solve the system: $y = 2x^2 + 1$
$3y = x$

Solution:

Write each equation as a function of x. Use a graphing calculator to graph the following system

$y_1 = 2x^2 + 1$

$y_2 = \frac{1}{3}x$



There are no intersection points, so the system has no solution. ■

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Name ______________________________ Date ______________

CHAPTER 9

Systems of Equations with at Least One Nonlinear Equation *continued*

EXAMPLE 4 Solve algebraically: no solution

Solve the system: $5x - x^2 = y$
$5x - y = -1$

Solution:

Substitute equation 1 into equation 2: $5x - (5x - x^2) = -1$

Solve for x: $x^2 = -1$

Since there is no number that can be squared to equal -1, there are no real number solutions to this equation. The system has no solution. ■

The third case is a system of equations with one solution. In other words, the graphs of the parabola and line intersect at only one point.

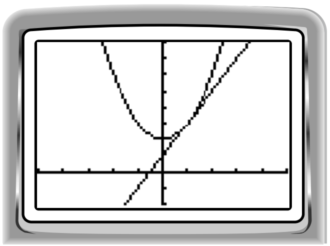
EXAMPLE 5 Solve by graphing: one solution

Solve the system: $y = x^2 + 2$
$y - 2x = 1$

Solution:

Write each equation as a function of x. Use a graphing calculator to graph the following system.

$y_1 = x^2 + 2$
$y_2 = 2x + 1$



There is one intersection point. Use the intersection option in the CALC menu of a graphing calculator to find the coordinates of the point of intersection.

The solution to the system is (1, 3). ■

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CHAPTER 9

Systems of Equations with at Least One Nonlinear Equation *continued*

EXAMPLE 6 Solve algebraically: one solution

Solve the system in Example 5 algebraically: $y = x^2 + 2$
$y - 2x = 1$

Solution:

Solve by substituting the first equation into the second.

$$y - 2x = 1$$
$$(x^2 + 2) - 2x = 1$$
$$x^2 - 2x + 1 = 0$$
$$(x - 1)^2 = 0$$
$$x - 1 = 0$$
$$x = 1$$

Substitute $x = 1$ into the first equation, $y = 1^2 + 2 = 3$.

Therefore, the solution set is (1, 3). ■

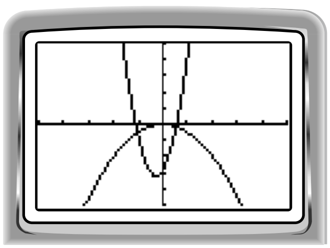
In the examples so far, you have looked at a system of a quadratic equation and a linear equation. This concept can be extended to systems of other equations such as circles and lines, circles and parabolas, two parabolas, and two circles.

EXAMPLE 7 Solve a system involving two parabolas: two solutions

Solve the system: $y = 4.5x^2 + 2.5x - 2.9$
$y = -0.5x^2$

Solution:

Use a graphing calculator to view the intersection points.



Use the intersection option in the CALC menu of the graphing calculator to approximate the intersection points.

Rounding to the nearest hundredth of a decimal place, the solutions to the system are (0.55, −0.15), (−1.05, −0.55). ■

Name ______________________ Date ____________

CHAPTER 9

Systems of Equations with at Least One Nonlinear Equation *continued*

EXAMPLE 8 Solve a system involving two parabolas: no solution

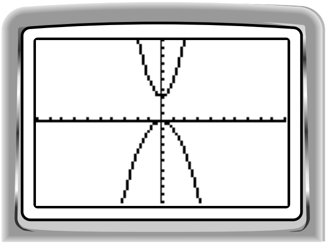
Solve the system: $y + x^2 = 0$
$y = 2x^2 + 3$

Solution:

Write each equation as a function of x. Use a graphing calculator to graph the following system.

$y_1 = -x^2$

$y_2 = 2x^2 + 3$



There are no intersection points, so the system has no solution. ■

Recall that an equation for a circle is given in the form $x^2 + y^2 = r^2$ where the center of the circle is at (0, 0) and the radius is r. Systems with a line and a circle or a parabola and a circle can be solved graphically or algebraically.

If you solve graphically using a graphing calculator, the equation of a circle $x^2 + y^2 = r^2$, will be entered as $y_1 = \sqrt{r^2 - x^2}$ and $y_2 = -\sqrt{r^2 - x^2}$.

EXAMPLE 9 Solve a system involving a circle and a parabola: three solutions

Solve the system: $x^2 + y^2 = 25$
$y = 5 - x^2$

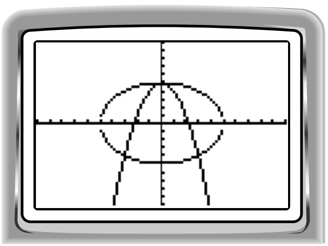
Solution:

The first equation represents a circle with center (0, 0) and radius 5. The second equation represents a parabola with vertex (0, 5) that points down. Graph the following on a graphing calculator.

$y_1 = \sqrt{25 - x^2}$

$y_2 = -\sqrt{25 - x^2}$

$y_3 = 5 - x^2$



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Name ______________________ Date ______________

CHAPTER 9

Systems of Equations with at Least One Nonlinear Equation *continued*

There appears to be three intersection points. Use the intersection option in the CALC menu of the graphing calculator to find the intersection points. This system has three solutions: (0, 5), (−3, −4), (3, −4). ■

Practice

Draw an example of the situation. If the situation is not possible, then write "not possible."

1. A line and a circle that have 1 intersection point.
2. Two circles that intersect in 4 places.
3. A parabola and a circle that have 4 intersection points.
4. A line and a circle that intersect in 3 places.

Solve the system algebraically. Do not use a graphing calculator.

5. $\begin{cases} y = 7x \\ x^2 + y = 0 \end{cases}$

6. $\begin{cases} x^2 + y^2 = 100 \\ y + 6 = 0 \end{cases}$

7. $\begin{cases} x^2 + y = 9 \\ x + y = -3 \end{cases}$

8. $\begin{cases} y = 2 - x^2 \\ y = x^2 - 4x + 4 \end{cases}$

Solve the system graphically. You may use a graphing calculator. Round your answer to the nearest hundredth.

9. $\begin{cases} x^2 + 6y = 12 \\ 4 - x^2 = y \end{cases}$

10. $\begin{cases} x^2 + y = 5.2 \\ y - x = 20 \end{cases}$

11. $\begin{cases} y = x^2 + 2x + 1 \\ y + 1 = -x^2 - 2x \end{cases}$

12. $\begin{cases} x^2 + y^2 = 4 \\ x^2 + y^2 = 7 \end{cases}$

13. $\begin{cases} y = \frac{1}{2}x - x^2 \\ x - y = 2 \end{cases}$

14. $\begin{cases} y - 2x^2 = 0.1x \\ y + x^2 = 1.5 \end{cases}$

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