9.7 Solve Systems with Quadratic Equations

Before You solved systems of linear equations.

Now You will solve systems that include a quadratic equation.

Why? So you can predict the path of a ball, as in Example 4.

You have solved systems of linear equations using the graph-and-check method and using the substitution method. You can use both of these techniques to solve a system of equations involving nonlinear equations, such as quadratic equations.

Recall that the substitution method consists of the following three steps.

STEP 1 **Solve** one of the equations for one of its variables.

STEP 2 **Substitute** the expression from Step 1 into the other equation and solve for the other variable.

STEP 3 **Substitute** the value from Step 2 into one of the original equations and solve.

COMMON CORE

CC.9-12.A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

EXAMPLE 1 Use the substitution method

Solve the system:

$y = 3x + 2$	Equation 1
$y = 3x^2 + 6x + 2$	Equation 2

Solution

STEP 1 **Solve** one of the equations for y. Equation 1 is already solved for y.

STEP 2 **Substitute** $3x + 2$ for y in Equation 2 and solve for x.

$y = 3x^2 + 6x + 2$	Write original Equation 2.
$3x + 2 = 3x^2 + 6x + 2$	Substitute $3x^2 + 2$ for y.
$0 = 3x^2 + 3x$	Subtract $3x$ and 2 from each side.
$0 = 3x(x + 1)$	Factor.
$3x = 0$ or $x + 1 = 0$	Zero-product property
$x = 0$ or $x = -1$	Solve for x.

AVOID ERRORS
Be sure to set all linear factors equal to zero when applying the zero-product property.

STEP 3 **Substitute** both 0 and −1 for x in Equation 1 and solve for y.

$y = 3x + 2$ $\quad$ $y = 3x + 2$

$y = 3(0) + 2$ $\quad$ $y = 3(-1) + 2$

$y = 2$ $\quad$ $y = -1$

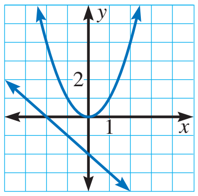
▶ The solutions are (0, 2) and (−1, −1).

POINTS OF INTERSECTION When you graph a system of equations, the graphs intersect at each solution of the system. For a system consisting of a linear equation and a quadratic equation the number of intersections, and therefore solutions, can be zero, one, or two.

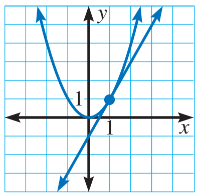
KEY CONCEPT *For Your Notebook*

Systems With One Linear Equation and One Quadratic Equation

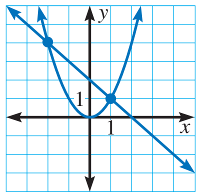
There are three possibilities for the number of points of intersection.



No Solution



One Solution



Two Solutions

EXAMPLE 2 Use a graphing calculator to solve a system

Solve the system: $y = 2x - 4$ **Equation 1**

$y = x^2 - 4x + 1$ **Equation 2**

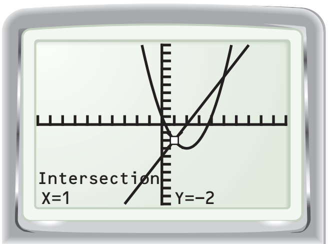
Solution

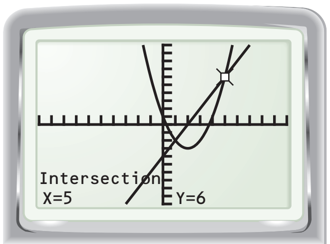
STEP 1 Enter each equation into your graphing calculator.

Set $Y_1 = 2x - 4$ and $Y_2 = x^2 - 4x + 1$.

STEP 2 Graph the system. Set a good viewing window. For this system, a good window is $-10 \le x \le 10$ and $-10 \le y \le 10$.

STEP 3 Use the *Trace* function to find the coordinates of each point of intersection. The points of intersection are (1, −2) and (5, 6).





▶ The solutions are (1, −2) and (5, 6).

CHECK Check the solutions. For example, check (1, −2).

$y = 2x - 4$ $\qquad$ $y = x^2 - 4x + 1$

$-2 \stackrel{?}{=} 2(1) - 4$ $\qquad$ $-2 \stackrel{?}{=} (1)^2 - 4(1) + 1$

$-2 = -2$ ✓ $\qquad$ $-2 = -2$ ✓

GUIDED PRACTICE for Examples 1 and 2

Solve the system of equations first by using the substitution method and then by using a graphing calculator.

1. $y = x + 4$
$y = 2x^2 - 3x - 2$

2. $y = x + 1$
$y = -x^2 + 6x + 1$

3. $y = x^2 - 6x + 11$
$y = x + 1$

SOLVING EQUATIONS You can use a graph to solve an equation in one variable. Treat each side of the equation as a function. Then graph each function on the same coordinate plane. The x-value of any points of intersection will be the solutions of the equation

EXAMPLE 3 Solve an equation using a system

Solve the equation $-x^2 - 4x + 2 = -2x - 1$ using a system of equations. Check your solution(s).

Solution

STEP 1 Write a system of two equations by setting both the left and right sides of the given equation each equal to y.

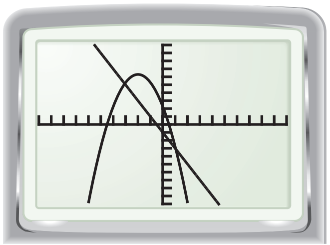
$-x^2 - 4x + 2 = -2x - 1$

$y = -x^2 - 4x + 2$ Equation 1

$y = -2x - 1$ Equation 2

AVOID ERRORS
If you draw your graph on graph paper, be very neat so that you can accurately identify any points of intersection.

STEP 2 Graph Equation 1 and Equation 2 on the same coordinate plane or on a graphing calculator.



STEP 3 The x-value of each point of intersection is a solution of the original equation. The graphs intersect at $(-3, 5)$ and $(1, -3)$.

▶ The solutions of the equation are $x = -3$ and $x = 1$.

CHECK: Substitute each solution in the original equation.

$$-x^2 - 4x + 2 = -2x - 1$$
$$-(-3)^2 - 4(-3) + 2 \stackrel{?}{=} -2(-3) - 1$$
$$-9 + 12 + 2 \stackrel{?}{=} 6 - 1$$
$$5 = 5 \checkmark$$

$$-x^2 - 4x + 2 = -2x - 1$$
$$-(1)^2 - 4(1) + 2 \stackrel{?}{=} -2(1) - 1$$
$$-1 - 4 + 2 \stackrel{?}{=} -2 - 1$$
$$-3 = -3 \checkmark$$

GUIDED PRACTICE for Example 3

Solve the equation using a system of equations.

4. $x + 3 = 2x^2 + 3x - 1$

5. $x^2 + 7x + 4 = 2x + 4$

6. $8 = x^2 - 4x + 3$

7. $-x + 4 = 3^x$

EXAMPLE 4 Solve a multi-step problem

BASEBALL During practice, you hit a baseball toward the gym, which is 240 feet away.

The path of the baseball after it is hit can be modeled by the equation:

$$y = -0.004x^2 + x + 3$$

The roof of the gym can be modeled by the equation:

$$y = \frac{2}{3}x - 120$$

for values of x greater than 240 feet and less than 320 feet.

The wall of the gym can be modeled by the equation:

$$x = 240$$

for values of y between 0 feet and 40 feet.

Does the baseball hit the roof of gym?

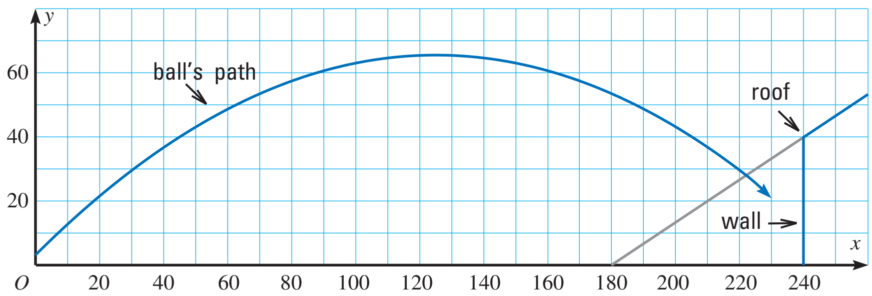
Solution

STEP 1 Write a system of two equations for the baseball and the roof.

$y = -0.004x^2 + x + 3$ **Equation 1 (baseball)**

$y = \frac{2}{3}x - 120$ **Equation 2 (roof)**

STEP 2 Graph both equations on the same coordinate plane.



STEP 3 The x-value where the graphs intersect is between 200 feet and 230 feet which is outside the domain of the equation for the roof.

▶ The baseball does not hit the roof.

✓ GUIDED PRACTICE for Example 4

8. **WHAT IF?** In Example 4, does the baseball hit the gym wall? If it does, how far up the wall does it hit? If it does not, how far away from the gym wall does the ball land?

9. **WHAT IF?** In Example 4, if you hit the ball so that it followed a path that had a smaller number as the coefficient of x^2, would it be *more* or *less* likely to hit the gym? *Explain*.

9.7 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 15, 19, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 28, and 35

SKILL PRACTICE

1. **VOCABULARY** Describe how to use the substitution method to solve a system of linear equations.

2. ★ **WRITING** Describe the possible number of solutions for a system consisting of a linear equation and a quadratic equation.

EXAMPLE 1 for Exs. 3–8

SUBSTITUTION METHOD **Solve the system of equations using the substitution method.**

3. $y = x^2 - x + 2$
   $y = x + 5$

4. $y = -x^2 + 4x - 2$
   $y = 4x - 6$

5. $y = x^2 - x$
   $y = -\frac{5}{2}x + 1$

6. $y = 2x^2 + x - 1$
   $y = -x - 3$

7. $y = 3x^2 - 6$
   $y = -3x$

8. $y = -2x^2 - 2x + 3$
   $y = \frac{7}{2}$

9. **ERROR ANALYSIS** Describe and correct the error in the solution steps shown.

$y = 3x^2 - 6x + 4$	Equation 1
$y = 4$	Equation 2
$y = 3(4)^2 - 6(4) + 4$	Substitute.
$y = 3(16) - 24 + 4 = 28$	Simplify

10. **COPY AND COMPLETE** When a system of equations includes a linear equation and a quadratic equation, there will (*always, sometimes, never*) be an infinite number of solutions.

11. ★ **MULTIPLE CHOICE** Which equation intersects the graph of $y = x^2 - 4x + 3$ twice?

    (A) $y = -1$ (B) $x = 2$

    (C) $y + 1 = x$ (D) $y + x = -1$

EXAMPLE 2 for Exs. 12–17

GRAPHING CALCULATOR **Use a graphing calculator to find the points of intersection, if any, of the graph of the system of equations.**

12. $y = 3x^2 - 2x + 1$
    $y = x + 7$

13. $y = x^2 + 2x + 11$
    $y = -2x + 8$

14. $y = -2x^2 - 4x$
    $y = 2$

15. $y = \frac{1}{2}x^2 - 3x + 4$
    $y = x - 2$

16. $y = \frac{1}{3}x^2 + 2x - 3$
    $y = 2x$

17. $y = 4x^2 + 5x - 7$
    $y = -3x + 5$

EXAMPLE 3 for Exs. 18–21

SOLVE THE EQUATION **Solve the equation using a system. Check each answer.**

18. $-5x + 5 = x^2 - 4x + 3$

19. $-6 = x^2 + 2x - 5$

20. $-3 = x^2 + 5x - 3$

21. $2x^2 + 4x = 2x + 4$

GRAPHING CALCULATOR **Use a graphing calculator to find the points of intersection, if any, of the graph of the system of equations.**

22. $y = -x^2 + 4$
$y = 5$

23. $y = -1$
$y = -2^x$

24. $y = x + 6$
$y = 0.5^x$

25. $y = -x^2 + 2x$
$y = -2x + 5$

26. $y = 3x - 1$
$y = 2^x$

27. $y = -1.5x + 1$
$y = 0.4^x$

28. ★ **WRITING** Describe the possible number of solutions for a system consisting of a quadratic equation and an exponential equation.

SOLVE THE EQUATION **Solve the equation using a system.**

29. $2^x + 1 = 2x + 1$

30. $4^x = -\frac{2}{3}x + 9$

31. $-2x + 11 = 2^x - 3$

32. $3^x - 5 = 6x - 8$

33. **CHALLENGE** Using a graphing calculator, find the points of intersection, if any, of the graphs of the equations $y = x^2 - 3x + 1$ and $y = x^2 - x - 1$. What are the solutions of the system?

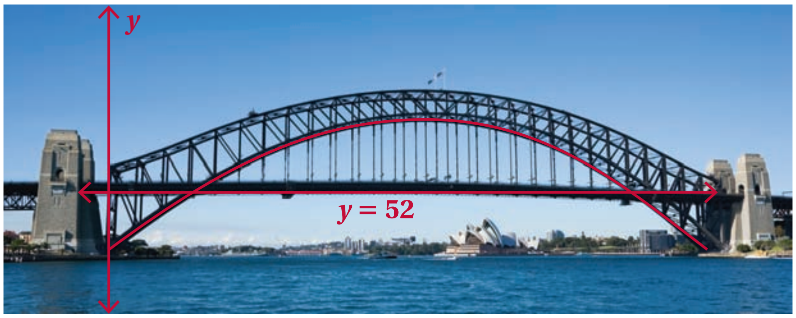
PROBLEM SOLVING

EXAMPLES 1 AND 2 for Exs. 34–36

34. **RECREATION** Marion and Reggie are driving boats on the same lake. Marion's chosen path can be modeled by the equation $y = -x^2 - 4x - 1$ and Reggie's path can be modeled by the equation $y = 2x + 8$. Do their paths cross each other? If so, what are the coordinates of the point(s) where the paths meet?

35. ★ **SHORT RESPONSE** Two dogs are running in a fenced dog park. One dog is following a path that can be modeled by the equation $y = 4$. Another dog is following a path that can be modeled by the equation $y = -x^2 + 3$. Will the dogs' paths cross? Explain your answer.

36. **ARCHITECTURE** The arch of the Sydney Harbor Bridge in Sydney, Australia, can be modeled by $y = -0.00211x^2 + 1.06x$ where x is the distance (in meters) from the left pylons and y is the height (in meters) of the arch above the water. The road can be modeled by the equation $y = 52$. To the nearest meter, how far from the left pylons are the two points where the road intersects the arch of the bridge?



○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

EXAMPLES 3 AND 4 for Exs. 37–39

37. **SAVINGS** Nancy and Miranda are looking at different ways to save money. Graph the two equations. Explain what happens when the graphs intersect. When will Miranda have more money saved than Nancy?

Nancy

I will save \$15 each month.
My money will not earn interest.

A model for my savings is $y = 15x$.

x is the number of months
y is my total savings

Miranda

I will put \$200 into an account that earns 2% annually. I will not save any more money.

If the interest is compounded monthly, $y = 200(1.02)^x$ models my savings.

x is months and y is my total savings.

38. **SPACE** Suppose an asteroid and a piece of space debris are traveling in the same plane in space. The asteroid follows a path that can be modeled locally by the equation $y = 2x^2 - 3x + 1$. The space debris follows a path that can be modeled locally by the equation $y = 8x - 13$.

    a. Will the paths of the two objects intersect? Is it possible for the two objects to collide? If so, what are the coordinates of the point where the paths intersect?

    b. What additional information would you need to decide whether the two objects will collide? *Explain*.

39. **MULTI-STEP PROBLEM** Keno asks Miguel if the graphs of all three of the equations shown below ever intersect in a single point.

    $y = 3x + 1 \qquad y = 2x^2 - 4x + 6 \qquad y = -2x + 6$

    a. Find any points of intersection of the graphs of $y = 3x + 1$ and $y = 2x^2 - 4x + 6$.

    b. Find any points of intersection of the graphs of $y = 3x + 1$ and $y = -2x + 6$.

    c. Find any points of intersection of the graphs $y = 2x^2 - 4x + 6$ and $y = -2x + 6$.

    d. Do the three graphs ever intersect in a single point? If so, what are the coordinates of this point of intersection?

40. **CHALLENGE** Find the point(s) of intersection, if any, for the line with equation $y = -x - 1$ and the circle with equation $x^2 + y^2 = 41$.