9.8 Compare Linear, Exponential, and Quadratic Models

Before You graphed linear, exponential, and quadratic functions.

Now You will compare linear, exponential, and quadratic models.

Why? So you can solve a problem about biology, as in Ex. 23.

Key Vocabulary
- linear function
- exponential function
- quadratic function

COMMON CORE

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

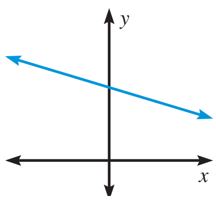
So far you have studied linear functions, exponential functions, and quadratic functions. You can use these functions to model data.

KEY CONCEPT — *For Your Notebook*

Linear, Exponential, and Quadratic Functions

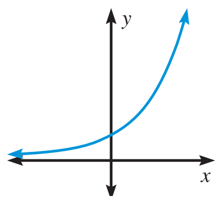
Linear Function

$y = mx + b$



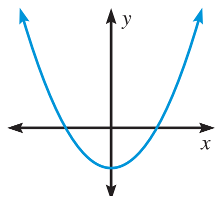
Exponential Function

$y = ab^x$



Quadratic Function

$y = ax^2 + bx + c$



EXAMPLE 1 Choose functions using sets of ordered pairs

Use a graph to tell whether the ordered pairs represent a *linear function*, an *exponential function*, or a *quadratic function*.

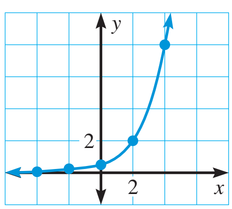
a. $\left(-4, \frac{1}{32}\right), \left(-2, \frac{1}{8}\right), \left(0, \frac{1}{2}\right), (2, 2), (4, 8)$

b. $(-4, 1), (-2, 2), (0, 3), (2, 4), (4, 5)$

c. $(-4, 5), (-2, 2), (0, 1), (2, 2), (4, 5)$

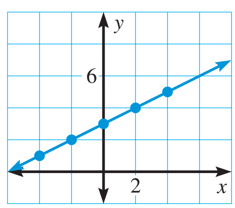
Solution

a.



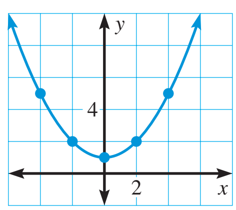
Exponential function

b.



Linear function

c.



Quadratic function

Animated Algebra at my.hrw.com

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DIFFERENCES AND RATIOS A table of values represents a linear function if the *differences* of successive y-values are all equal. A table of values represents an exponential function if the *ratios* of successive y-values are all equal. In both cases, the increments between successive x-values need to be equal.

Linear function: $y = 3x + 5$

x	−1	0	1	2
y	2	5	8	11

Differences: $5 - 2 = 3$ 3   3

Exponential function: $y = 0.5(2)^x$

x	−1	0	1	2
y	0.25	0.5	1	2

Ratios: $\frac{0.5}{0.25} = 2$ 2   2

ANALYZE FIRST DIFFERENCES
The first differences of a linear function are constant, which is not the case for quadratic functions or exponential functions. For instance, the first differences for the y-values shown for $y = 0.5(2)^x$ are $0.5 - 0.25 = 0.25$, $1 - 0.5 = 0.5$, and $2 - 1 = 1$.

You can use differences to tell whether a table of values represents a quadratic function, as shown.

Quadratic function: $y = x^2 - 2x + 2$

x	−1	0	1	2	3
y	5	2	1	2	5

First differences: −3   −1   1   3

Second differences: 2   2   2

First find the differences of successive y-values, or *first differences*.

Then find the differences of successive first differences, or *second differences*.

The table of values represents a quadratic function if the second differences are all equal.

EXAMPLE 2 Identify functions using differences or ratios

Use differences or ratios to tell whether the table of values represents a *linear function*, an *exponential function*, or a *quadratic function*. Extend the table to find the y-value for the next x-value.

a.

x	−2	−1	0	1	2
y	−6	−6	−4	0	6

First differences: 0   2   4   6

Second differences: 2   2   2

▶ The table of values represents a quadratic function. When $x = 3$, $y = 6 + 8 = 14$.

b.

x	−2	−1	0	1	2
y	−2	1	4	7	10

Differences: 3   3   3   3

▶ The table of values represents a linear function. When $x = 3$, $y = 10 + 3 = 13$.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Tell whether the ordered pairs represent a *linear function*, an *exponential function*, or a *quadratic function*: (0, −1.5), (1, −0.5), (2, 2.5), (3, 7.5).

2. Tell whether the table of values represents a *linear function*, an *exponential function*, or a *quadratic function*.

x	−2	−1	0	1
y	0.08	0.4	2	10

WRITING AN EQUATION When you decide that a set of ordered pairs represents a linear, an exponential, or a quadratic function, you can write an equation for the function. In this lesson, when you write an equation for a quadratic function, the equation will have the form $y = ax^2$.

EXAMPLE 3 Write an equation for a function

Tell whether the table of values represents a *linear function*, an *exponential function*, or a *quadratic function*. Then write an equation for the function.

x	−2	−1	0	1	2
y	2	0.5	0	0.5	2

Solution

STEP 1 **Determine** which type of function the table of values represents.

x	−2	−1	0	1	2
y	2	0.5	0	0.5	2

First differences: −1.5 −0.5 0.5 1.5

Second differences: 1 1 1

The table of values represents a quadratic function because the second differences are equal.

STEP 2 **Write** an equation for the quadratic function. The equation has the form $y = ax^2$. Find the value of a by using the coordinates of a point that lies on the graph, such as (1, 0.5).

$y = ax^2$ **Write equation for quadratic function.**

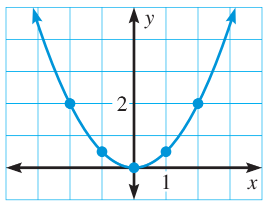
$0.5 = a(1)^2$ **Substitute 1 for x and 0.5 for y.**

$0.5 = a$ **Solve for a.**

▸ The equation is $y = 0.5x^2$.

AVOID ERRORS
In Example 3, do not use (0, 0) to find the value of a, even though (0, 0) lies on the graph of $y = ax^2$. If you do, you will obtain an undefined value for a.

CHECK Plot the ordered pairs from the table. Then graph $y = 0.5x^2$ to see that the graph passes through the plotted points.



✓ GUIDED PRACTICE for Example 3

Tell whether the table of values represents a *linear function*, an *exponential function*, or a *quadratic function*. Then write an equation for the function.

3.

x	−3	−2	−1	0	1
y	−7	−5	−3	−1	1

4.

x	−2	−1	0	1	2
y	8	2	0	2	8

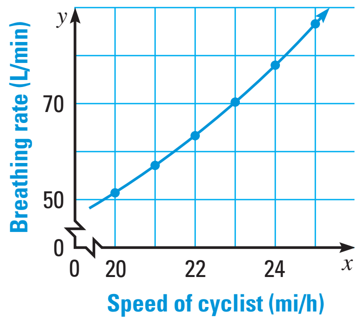
EXAMPLE 4 Solve a multi-step problem

CYCLING The table shows the breathing rates y (in liters of air per minute) of a cyclist traveling at different speeds x (in miles per hour). Tell whether the data can be modeled by a *linear function*, an *exponential function*, or a *quadratic function*. Then write an equation for the function.

Speed of cyclist, x (mi/h)	20	21	22	23	24	25
Breathing rate, y (L/min)	51.4	57.1	63.3	70.3	78.0	86.6

Solution

STEP 1 **Graph** the data. The graph has a slight curve. So, a linear function does not appear to model the data.



STEP 2 **Decide** which function models the data. In the table below, notice that $\frac{57.1}{51.4} \approx 1.11$, $\frac{63.3}{57.1} \approx 1.11$, $\frac{70.3}{63.3} \approx 1.11$, $\frac{78.0}{70.3} \approx 1.11$, and $\frac{86.6}{78.0} \approx 1.11$. So, the ratios are all approximately equal. An exponential function models the data.

Speed of cyclist, x (mi/h)	20	21	22	23	24	25
Breathing rate, y (L/min)	51.4	57.1	63.3	70.3	78.0	86.6
Ratios:		1.11	1.11	1.11	1.11	1.11

REVIEW EXPONENTIAL FUNCTIONS
You may want to review writing an equation for an exponential function.

STEP 3 **Write** an equation for the exponential function. The breathing rate increases by a factor of 1.11 liters per minute, so $b = 1.11$. Find the value of a by using one of the data pairs, such as (20, 51.4).

$y = ab^x$ **Write equation for exponential function.**

$51.4 = a(1.11)^{20}$ **Substitute 1.11 for *b*, 20 for *x*, and 51.4 for *y*.**

$\frac{51.4}{(1.11)^{20}} = a$ **Solve for *a*.**

$6.38 \approx a$ **Use a calculator.**

▶ The equation is $y = 6.38(1.11)^x$.

✓ GUIDED PRACTICE for Example 4

5. In Example 4, suppose the cyclist is traveling at 15 miles per hour. Find the breathing rate of the cyclist at this speed.

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9.8 EXERCISES

HOMEWORK KEY

◯ = See **WORKED-OUT SOLUTIONS** Exs. 7, 13, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 26, and 27

◆ = **MULTIPLE REPRESENTATIONS** Ex. 25

SKILL PRACTICE

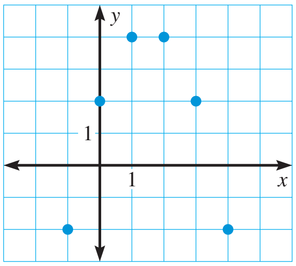
1. **VOCABULARY** Copy and complete: A function that is of the form $y = ab^x$ is a(n) __?__.

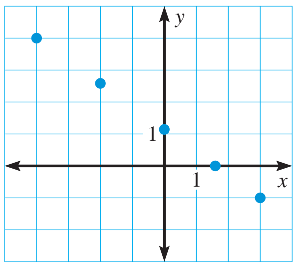
2. ★ **WRITING** *Describe* how you can tell whether a table of values represents a quadratic function.

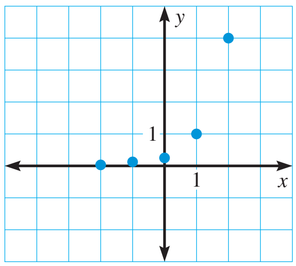
EXAMPLE 1 for Exs. 3–11

MATCHING **Match the function with the graph that the function represents.**

3. Linear function
4. Exponential function
5. Quadratic function

A.


B.


C.


USING A GRAPH **Use a graph to tell whether the ordered pairs represent a *linear function*, an *exponential function*, or a *quadratic function*.**

6. $(-4, -7), (-2, -1), (0, 1), (2, -1), (4, -7)$

7. $(-5, -1), (-3, 0), (-1, 1), (1, 2), (3, 3)$

8. $\left(-1, \frac{1}{16}\right), \left(0, \frac{1}{4}\right), (1, 1), (2, 4), (3, 16)$

9. $(-1, 8), (1, 2), \left(3, \frac{1}{2}\right), \left(5, \frac{1}{8}\right), \left(7, \frac{1}{32}\right)$

10. $(-4, -4), (-2, -3.5), (0, -3), (2, -2.5)$

11. $(-1, 0.5), (0, -0.5), (1, 0.5), (2, 3.5)$

EXAMPLES 2 and 3 for Exs. 12–19

USING DIFFERENCES AND RATIOS **Tell whether the table of values represents a *linear function*, an *exponential function*, or a *quadratic function*. Then write an equation for the function.**

12.

x	0	1	2	3	4
y	1	0	−1	−2	−3

13.

x	−2	−1	0	1	2
y	−4	−1	0	−1	−4

14.

x	−3	−2	−1	0	1
y	13.5	6	1.5	0	1.5

15.

x	−2	−1	0	1	2
y	−5	−2	1	4	7

16.

x	−2	−1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

17.

x	−1	0	1	2	3
y	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$

18. ★ **MULTIPLE CHOICE** Which function is represented by the following ordered pairs: $(-1, 4), (0, 0), (1, 4), (2, 16), (3, 36)$?

Ⓐ $y = 0.25x^2$ Ⓑ $y = 4^x$ Ⓒ $y = 4x^2$ Ⓓ $y = 4x$

19. ERROR ANALYSIS *Describe* and correct the error in writing an equation for the function represented by the ordered pairs.

(0, 0), (1, 2.5), (2, 10), (3, 22.5), (4, 40)

x	0	1	2	3	4
y	0	2.5	10	22.5	40

First differences: 2.5 7.5 12.5 17.5

Second differences: 5 5 5

The ordered pairs represent a quadratic function.

$y = ax^2$

$2 = a(10)^2$

$0.02 = a$

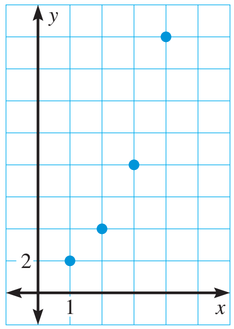
So, the equation is $y = 0.02x^2$.

20. REASONING Use the graph shown.

a. Tell whether the graph represents an *exponential function* or a *quadratic function* by looking at the graph.

b. Make a table of values for the points on the graph. Then use differences or ratios to check your answer in part (a).

c. Write an equation for the function that the table of values from part (b) represents.



21. GEOMETRY The table shows the area A (in square centimeters) of an equilateral triangle for various side lengths s (in centimeters). Write an equation for the function that the table of values represents. Then find the area of an equilateral triangle that has a side length of 10 centimeters.

Side length, s (cm)	1	2	3	4	5
Area, A (cm^2)	$0.25\sqrt{3}$	$\sqrt{3}$	$2.25\sqrt{3}$	$4\sqrt{3}$	$6.25\sqrt{3}$

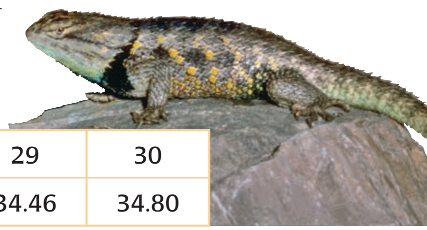
22. CHALLENGE In the ordered pairs below, the y-values are given in terms of m. Tell whether the ordered pairs represent a *linear function*, an *exponential function*, or a *quadratic function*.

$(1, 3m - 1), (2, 10m + 2), (3, 26m), (4, 51m - 7), (5, 85m - 19)$

PROBLEM SOLVING

EXAMPLE 4 for Exs. 23–25

23. LIZARDS The table shows the body temperature B (in degrees Celsius) of a desert spiny lizard at various air temperatures A (in degrees Celsius). Tell whether the data can be modeled by a *linear function*, an *exponential function*, or a *quadratic function*. Then write an equation for the function.



Air temperature, A (°C)	26	27	28	29	30
Body temperature, B (°C)	33.44	33.78	34.12	34.46	34.80

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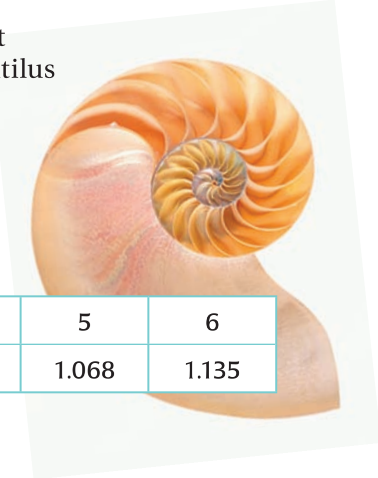
24. NAUTILUS A chambered nautilus is a marine animal that lives in the outermost chamber of its shell. When the nautilus outgrows a chamber, it adds a new, larger chamber to its shell. The table shows the volumes (in cubic centimeters) of consecutive chambers of a nautilus. Tell whether the data can be modeled by a *linear function*, an *exponential function*, or a *quadratic function*. Then write an equation for the function.



Chamber	1	2	3	4	5	6
Volume (cm^3)	0.836	0.889	0.945	1.005	1.068	1.135

25. **MULTIPLE REPRESENTATIONS** In 1970, the populations of Troy and Union were each 3000. From 1970 to 2010, the population of Troy doubled every decade. The population of Union increased by 3000 every decade. For each town, consider the set of data pairs (n, P) where n is the number of decades since 1970 and P is the population.

a. Describing in Words Tell whether the data for each town can be modeled by a *linear function*, an *exponential function*, or a *quadratic function*. *Explain* your reasoning.

b. Making a Table Make a table for each town. Use the tables to justify your answers in part (a).

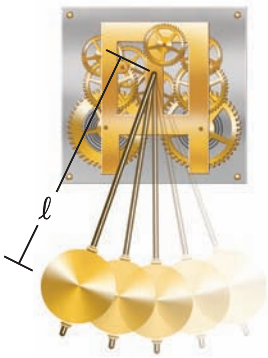
c. Writing a Model For each town, do the following: Write an equation for the function that models the data. Suppose that the growth pattern continues. *Predict* the population in 2030.

26. ★ **MULTIPLE CHOICE** The table shows the cost of a custom circular rug for various diameters (in feet). What is the approximate cost of a custom circular rug that has a diameter of 8 feet?

Diameter (ft)	2	3	4	5	6
Cost (dollars)	28.40	63.90	113.60	177.50	255.60

(A) \$333.70 (B) \$411.80 (C) \$454.40 (D) \$908.80

27. ★ **EXTENDED RESPONSE** The time it takes for a clock's pendulum to swing from one side to the other and back again, as shown in the back view of the clock, is called the pendulum's period. The table shows the period t (in seconds) of a pendulum of length ℓ (in feet).



Period, t (sec)	1	2	3	4	5
Length, ℓ (ft)	0.82	3.28	7.38	13.12	20.5

a. Model Tell whether the data can be modeled by a *linear function*, an *exponential function*, or a *quadratic function*. Then write an equation for the function.

b. Apply Find the length of a pendulum that has a period of 0.5 second.

c. Analyze How does decreasing the length of the pendulum by 50% change the period? *Justify* your answer using several examples.

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◯ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE** = **MULTIPLE REPRESENTATIONS**

28. **CHALLENGE** The table shows the height h (in feet) that a pole vaulter's center of gravity reaches for various running speeds s (in feet per second) at the moment the pole vaulter launches himself into the air.

Running speed, *s* (ft/sec)	30	31	32	33	34
Height of center of gravity, *h* (ft)	$14\frac{1}{16}$	$15\frac{1}{64}$	16	$17\frac{1}{64}$	$18\frac{1}{16}$

a. A pole vaulter is running at $31\frac{1}{2}$ feet per second when he launches himself into the air. Find the height that the pole vaulter's center of gravity reaches.

b. Find the speed at which the pole vaulter needs to be running when he launches himself into the air in order for his center of gravity to reach a height of 19 feet. Round your answer to the nearest foot per second.