Name ______________________ Date ______________

CHAPTER 9

Compare Linear, Exponential and Quadratic Graphs

It is interesting to compare and contrast some additional features of the graphs of linear, exponential and quadratic functions.

KEY CONCEPT

Increasing and Decreasing Functions

A function is said to be **always increasing** if given any two x-values a and b where $b > a$, the corresponding y-value at b is greater than the corresponding y-value at a. A function is said to be **increasing over an interval** if given any two x-values a and b in the interval where $b > a$, the corresponding y-value at b is greater than the corresponding y-value at a.

A function is said to be **always decreasing** if given any two x-values a and b where $b > a$, the corresponding y-value at b is less than the corresponding y-value at a. A function is said to be **decreasing over an interval** if given any two x-values a and b in the interval where $b > a$, the corresponding y-value at b is less than the corresponding y-value at a.

EXAMPLE 1

Features of $y = mx + b$

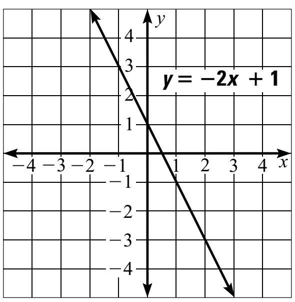
Show that the function $y = -2x + 1$ has a constant rate of change. Then determine where the function is decreasing and/or increasing.

Solution:

The rate of change between any two real numbers a and b ($b > a$) is:

$$\frac{(-2b + 1) - (-2a + 1)}{b - a} = \frac{-2b + 2a}{b - a} = \frac{-2(b - a)}{b - a} = -2.$$

This shows that the rate of change is constant for all values of x. The graph of the function confirms that the rate of change is constant.



The graph is *always decreasing*. ■

If a function is increasing over an interval, then rates of change within that interval will always be positive. Similarly, if a function is decreasing over an interval, rates of change within that interval will always be negative. Portions of a graph that are straight lines will have a constant rate of change.

Name ______________________ Date ____________

CHAPTER 9

Compare Linear, Exponential and Quadratic Graphs *continued*

EXAMPLE 2 Features of $y = ab^x$

Show that the exponential function $y = 2 \cdot 3^x$ has a variable rate of change. Then determine where the function is decreasing and/or increasing.

Solution:

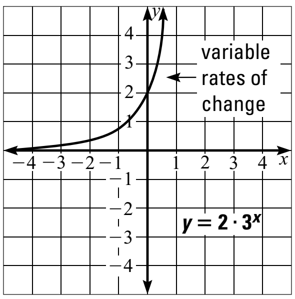
Compute the rate of change from $x = 0$ to $x = 1$:

$$\frac{2 \cdot 3^1 - 2 \cdot 3^0}{1 - 0} = \frac{6 - 2}{1} = 4$$

Now, compute the rate of change from $x = 1$ to $x = 2$:

$$\frac{2 \cdot 3^2 - 2 \cdot 3^1}{2 - 1} = \frac{18 - 6}{1} = 12$$

This shows that the rate of change is variable. The graph of the function confirms that the rate of change is variable.



The graph is *always increasing.* ■

EXAMPLE 3 Features of $y = ax^2 + bx + c$

Show that the quadratic function $y = x^2 + 4x + 4$ has a variable rate of change. Then determine where the function is decreasing and/or increasing.

Solution:

Compute the rate of change between $x = -3$ and $x = -2$:

$$\frac{\left[(-2)^2 + 4(-2) + 4\right] - \left[(-3)^2 + 4(-3) + 4\right]}{-2 - (-3)} = \frac{0 - 1}{1} = -1$$

Compute the rate of change between $x = -2$ and $x = -1$.

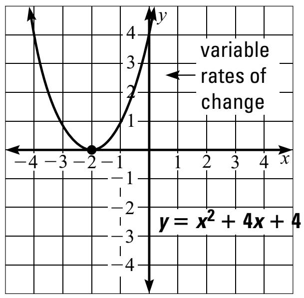
$$\frac{\left[(-1)^2 + 4(-1) + 4\right] - \left[(-2)^2 + 4(-2) + 4\right]}{-1 - (-2)} = \frac{1 - 0}{1} = 1$$

Name ______________________ Date ______________

CHAPTER 9

Compare Linear, Exponential and Quadratic Graphs *continued*

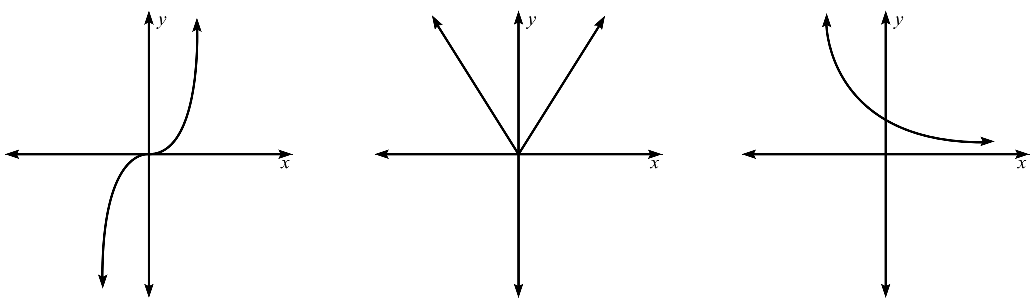
This shows that the rate of change is variable. The graph of the function confirms that the rate of change is variable.



Notice that the graph is both *decreasing* and *increasing*. The vertex (−2, 0) is where the graph changes from decreasing to increasing. The graph is decreasing on the interval $x < -2$, and increasing on the interval $x > 2$. ■

Practice

Use the graphs below to answer Exercises 1–3.



Graph A Graph B Graph C

1. Compare the graph of $y = x^2$ to Graph B. Compare in terms of increasing and/or decreasing, symmetry, rates of change, extreme points, and other features. Be specific.

2. Compare the graph of $y = 2x$ to graph A. Compare in terms of increasing and/or decreasing, symmetry, rates of change, extreme points, and other features.. Be specific.

3. Compare the graph of $y = 2^x$ to graph C. Compare in terms of increasing and/or decreasing, symmetry, rates of change, extreme points, and other features. Be specific.