9.9 Model Relationships

Before You studied linear, exponential, and quadratic functions.

Now You will compare representations of these functions.

Why So you can model the height of water, as in Example 1.

Key Vocabulary
- verbal model
- slope
- vertex

Sometimes you will find it helpful to model a function with a graph even if you don't have enough information to write an equation to model the function.

Sketching a graph based on a description of a situation can help you understand the situation and identify key features of the model.

COMMON CORE

CC.9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*

EXAMPLE 1 Sketch a graph of a real-world situation

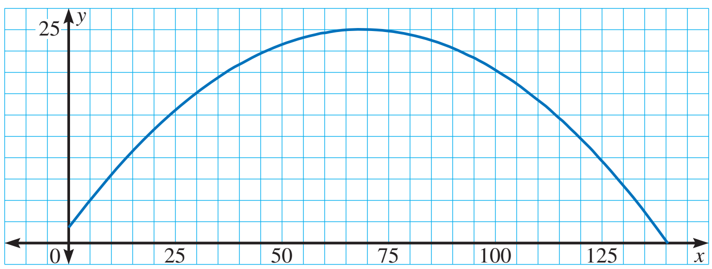
FIRE-FIGHTING The water from one water cannon on a fire-fighting boat reaches a maximum height of 25 feet and travels a horizontal distance of about 140 feet.

a. What type of function should you use to represent the path of the water? Sketch a graph of the path of the water.

b. In the context of the given situation, what do the intercepts and maximum point represent?

Solution

a. The path of the water can be modeled by a parabola. Let x represent the horizontal distance in feet and let y represent the vertical distance in feet.



b. Because the water cannon is on a boat, the graph has only one x-intercept where the water reaches the surface of the water or ground. The maximum point of the graph is where the water reaches its maximum height, about 70 feet from the boat.

GUIDED PRACTICE for Example 1

1. Using the graph in Example 1, describe the intervals in which the function is increasing and decreasing. Explain what the intervals mean in the given situation.

EXAMPLE 2 Compare properties of two linear functions

Decide which linear function is increasing at a greater rate.

- Linear Function 1 has an x-intercept of 4 and a y-intercept of -2.
- Linear Function 2 includes the points in the table below.

x	-2	-1	0	1	2	3
y	-11	-6	-1	4	9	14

AVOID ERRORS
In calculating the slope of a linear function, remember to divide the change in y by the change in x.

Solution

The slope of a linear equation indicates how rapidly a linear function is increasing or decreasing. The points (4, 0) and (0, -2) are on the graph of Linear Function 1, so its slope is $\frac{0-(-2)}{4-0} = \frac{1}{2}$.

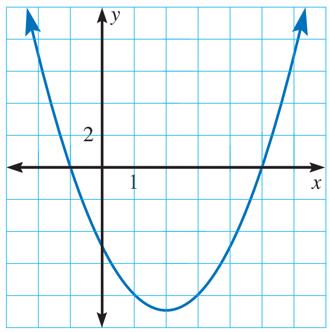
The table for Linear Function 2 shows that for each increase of 1 in the value of x there is an increase of 5 in the value of y, so its slope is $\frac{5}{1} = 5$.

▶ Linear Function 2 is increasing more rapidly.

EXAMPLE 3 Compare properties of two quadratic functions

Use the given information to decide which quadratic function has the lesser minimum value.

- Quadratic Function 1: The function whose equation is $y = 3x^2 - 12x + 1$.
- Quadratic Function 2: The function whose graph is shown at the right.



STUDY HELP
Review the lesson *Graph* $y = ax^2 + bx + c$ for information on finding the coordinates of the minimum value of a quadratic function.

Solution

The minimum value of Quadratic Function 1 is the y-value of the vertex of its parabola. The x-coordinate of the vertex is $-\left(\frac{b}{2a}\right) = -\frac{-12}{2(3)} = \frac{12}{6} = 2$. When $x = 2$, $y = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$. So the vertex is (2, -11) and the minimum value is -11.

The minimum value of Quadratic Function 2 can be seen on the graph of the function; it is -9.

▶ Quadratic Function 1 has the lesser minimum value.

✓ GUIDED PRACTICE for Examples 2 and 3

2. **COMPARE** Compare the rates of change in the linear functions $y = 4x + 5$ and $y = 3 - 4x$.

3. **WHAT IF?** In Example 3, replace the equation for Quadratic Function 1 with $y = x^2 - 6x - 7$. Which function now has the lesser minimum value?

EXAMPLE 4 Choose a model for a real-world situation

BUSINESS The table shows the revenue generated by a company during each of the previous five years. Based on the change per unit interval, choose an appropriate type of function to model the situation.

Year	2008	2009	2010	2011	2012
Revenue ($)	50,000	51,500	53,045	54,636	56,275

Solution

The revenue is increasing each year by about 3%. Because the quantity grows by a constant percent rate per unit interval, you should use an exponential growth model for the situation.

EXAMPLE 5 Choose a model for a real-world situation

FURNITURE You are a furniture salesperson and earn $200 a week plus a 5% commission on the total value of all sales you make during the week.

a. Based on the given information, choose an appropriate type of function to model your potential weekly earnings as a function of sales.

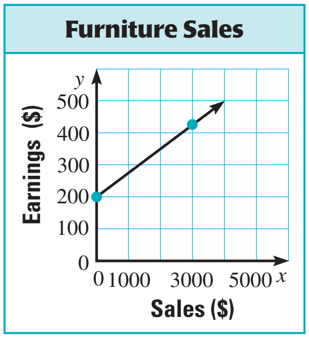
b. Sketch a graph representing your potential earnings for any given week as a function of sales. Identify the function's intercept(s) and interpret the meaning of each intercept in the context of the given situation.

Solution

a. For every $100 of sales, your earnings increase by $5. Earnings are increasing by a constant rate. Use a linear function.

b. Let x represent the weekly sales and let y represent total earnings. The y-intercept is 200 and represents your weekly salary when you do not sell any furniture during that week. The function only makes sense for $x \geq 0$, so there is no x-intercept.

STUDY HELP
Remember that the graph of a real-world function does not necessarily have both an x-intercept and a y-intercept.



✓ GUIDED PRACTICE for Examples 4 and 5

4. RUNNING The table shows the distance that Juan covered per hour in the first four hours of a triathlon. Based on the change per unit interval, choose an appropriate function to model the situation.

Hour	1	2	3	4
Miles	6	5.4	4.86	4.374

9.9 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 3, 5, 11, and 15

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 11, 18, and 19

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A ___?___ describes a real-world situation using words as labels and using math symbols to relate the words.

2. ★ **WRITING** Explain the relationship between the slope of a linear function and the concept of an increasing/decreasing linear function.

EXAMPLE 1 for Exs. 3–5

3. **CHOOSE A MODEL** A hot air balloon has already risen 20 feet above the ground. At this point in time it begins to rise at a steady rate of 2 feet per second.

   a. What type of function would be a good model for this situation?

   b. Sketch a graph representing the balloon's altitude y in terms of the time x since it resumed its ascent.

   c. Identify the intervals on which the graph is increasing or decreasing and explain what these intervals mean in the context of the situation.

4. **CHOOSE A MODEL** During a weekend trip riding his motorcycle, Neil plans to average 55 miles per hour.

   a. What type of function would be a good model for this situation?

   b. Sketch a graph representing the distance he will travel y in terms of the number of hours x that he rides.

   c. Use the graph to identify the intercept(s) and interpret the meaning of each intercept in the context of the situation.

5. **CHOOSE A MODEL** A juggler throws a ball into the air. It reaches a maximum height of about 25 feet, and the juggler catches it again after 2.5 seconds.

   a. What type of function would be a good model for this situation?

   b. Sketch the graph of an equation that could model the height of the ball as a function of time.

   c. Identify the intervals on which the graph is increasing or decreasing and explain what these intervals mean in the context of the situation.

6. ★ **MULTIPLE CHOICE** Marvin is making a rectangular quilt. Suppose the width of the quilt is x meters and length of the quilt is $(3 - x)$ meters. Which type of function should you use to model the area y of the quilt in terms of its width?

   (A) linear　　(B) quadratic

   (C) exponential growth　　(D) exponential decay

ERROR ANALYSIS **In Exercises 7 and 8, *describe* and correct the error in the model.**

7. A student uses a quadratic function to model a population that is increasing by 4% per year.

8. A student uses an exponential decay function to model the distance traveled over time of a car traveling at a steady speed of 50 miles per hour.

EXAMPLES 2 and 3 for Exs. 9–10

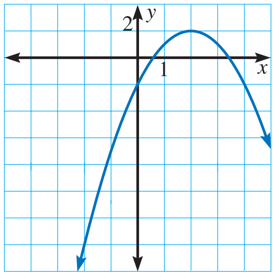
9. LINEAR FUNCTIONS Use the given information to decide which linear function is decreasing more rapidly.

- Linear Function 1 has a y-intercept of -3 and a slope of -1.
- The table shows the coordinates of six points found on the line representing Linear Function 2.

x	-4	-2	0	2	4	6
y	8	4	0	-4	-8	-12

10. QUADRATIC FUNCTIONS Use the given information to decide which quadratic function has the greater maximum value.

- Quadratic Function 1: The function whose equation is $y = -x^2 + 4x + 2$.
- Quadratic Function 2: The function whose graph is shown at the right.



EXAMPLES 4 and 5 for Exs. 11–12

11. ★ **MULTIPLE CHOICE** Decide which relationship grows by a constant percent rate per unit interval.

(A)

Hours	1	2	3	4
Miles	5	10	15	20

(B)

Year	1	2	3	4
Revenue ($)	2000	1500	1125	843.75

(C)

Days	1	2	3	4
Windows Installed	2	4	8	16

(D)

Minutes	1	2	3	4
Pancakes Made	27	9	3	1

12. CHALLENGE In 2010, the United States Census Bureau estimated that there were approximately 310 million people in the United States. By some estimates the population is growing by about 0.9% per year.

a. Sketch a graph relating the population y at any time x. Let $x = 0$ represent the year 2010. What type of function will be a good model?

b. Interpret the meaning of the x- and y-intercepts, if they exist, in terms of the context of this situation.

c. Is the graph of the function increasing or decreasing? *Explain.*

◯ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

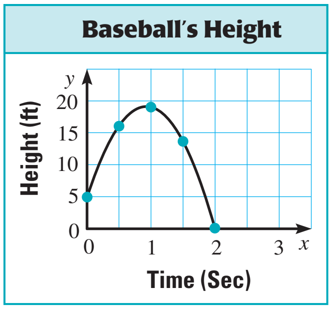
PROBLEM SOLVING

EXAMPLES 3 and 4 for Exs. 13–15

13. MUSIC Celia has already downloaded 14 songs to her cell phone. In the future she intends to download 2 songs per week. Her friend Connie has already downloaded 12 songs to her cell phone and she plans to download songs based on the table below. Which girl's playlist is growing faster?

Week	1	2	3	4
Total Number of Songs on Connie's Cell Phone	17	22	27	32

14. BASEBALL Tim threw a baseball in the air. Suppose the ball's height in feet can be modeled by the equation $y = -16x^2 + 40x + 5$. Matt threw the same baseball in the air. The graph models the height in feet of Matt's ball as a function of time. Which ball reached a greater height?



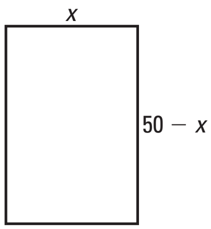
15. SCIENCE Tanya placed mold spores in a Petri dish. The table shows the number of spores in the dish at the end of each hour. Indicate whether the number of spores in the Petri dish represents *growth*, *decay*, or *neither*. Identify the growth or decay rate, if it exists, expressing it as a percent.

Hour	1	2	3	4
Number of Spores	16	24	36	54

16. CHICKENS You have 100 meters of fencing to build a chicken pen.

a. Use the diagram to help sketch a graph representing the area y of the pen in terms of the width x of the pen.

b. Use the graph to identify the intercept(s) and interpret the meaning of each intercept in the context of the situation.



17. ROWING The table shows the distance in miles that a rowing crew covered during each 15-minute interval in the first hour of practice. Based on the change per unit interval, choose an appropriate function to model the situation.

Minutes	15	30	45	60
Miles	2	1.6	1.28	1.024

18. ★ **MULTIPLE CHOICE** Choose the situation in which one quantity changes by a constant amount per unit interval relative to a second quantity.

(A) Michael rented tables and chairs for a party. The cost for the rental was $10 for the first day. If he keeps them for more than one day the cost per day is double the preceding day.

(B) Alexi's stamp collection already contains 12 stamps. Each time he goes to the post office he will buy 2 stamps to add to his collection.

(C) Sue has an ant farm. The number of ants can be modeled by the equation $y = 100(1.05)^x$, where y is the number of ants on any given day and x is the number of days since she started the farm.

(D) Pam accidently dropped her watch from her tree house. The equation $y = -16x^2 + 37$ models the height of the watch as it is falling to the ground. The variable y represents the height of the watch and x represents the time in seconds since Pam dropped the watch.

19. ★ **EXTENDED RESPONSE** Use the information to answer each question.

a. Graphs Using a single coordinate system, graph the functions $y = 2x$, $y = x^2$, and $y = 2^x$. Which function eventually has the greatest y-value for a given value of x?

b. Tables Complete a table similar to the one below for each of the three given functions. Which function eventually has the greatest y-value for a given value of x?

Linear Function: $y = 3x + 1$
Quadratic Function: $y = 3x^2 + 1$
Exponential Function: $y = 3^x + 1$

x	−1	0	2	4	6	8	10
y	?	?	?	?	?	?	?

c. CHALLENGE Given any quantity that can be modeled by a linear function, any quantity that can be modeled by a quadratic function, and any quantity that can be modeled by an exponential growth function, can you predict which quantity will eventually exceed the other two? *Explain*.

QUIZ

1. Tell whether the table of values represents a *linear function*, an exponential *function*, or a *quadratic function*. Then write an equation for the function.

x	1	2	3	4	5
y	5	1	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{125}$

2. Solve the system shown. $y = x^2 - 4x - 11$
$y = x - 5$

3. **EXERCISE** You ride your bicycle 2 miles to a local park and join friends for a hike. You walk at a rate of 3 miles per hour. Sketch a graph that relates total distance y traveled to time x walking. Identify the intercept(s). Interpret them in the context of the situation.

See EXTRA PRACTICE in Student Resources