1.2 Apply Order of Operations

Before	You evaluated algebraic expressions and used exponents.
Now	You will use the order of operations to evaluate expressions.
Why?	So you can determine online music costs, as in Ex. 35.

Key Vocabulary
• order of operations

Mathematicians have established an **order of operations** to evaluate an expression involving more than one operation.

KEY CONCEPT   For Your Notebook

Order of Operations

STEP 1 **Evaluate** expressions inside grouping symbols.

STEP 2 **Evaluate** powers.

STEP 3 **Multiply** and **divide** from left to right.

STEP 4 **Add** and **subtract** from left to right.

EXAMPLE 1   Evaluate expressions

Evaluate the expression $27 \div 3^2 \times 2 - 3$.

STEP 1 There are no grouping symbols, so go to Step 2.

STEP 2 **Evaluate** powers.

$27 \div 3^2 \times 2 - 3 = 27 \div 9 \times 2 - 3$ **Evaluate power.**

STEP 3 **Multiply** and **divide** from left to right.

$27 \div 9 \times 2 - 3 = 3 \times 2 - 3$ **Divide.**

$3 \times 2 - 3 = 6 - 3$ **Multiply.**

STEP 4 **Add** and **subtract** from left to right.

$6 - 3 = 3$ **Subtract.**

▶ The value of the expression $27 \div 3^2 \times 2 - 3$ is 3.

✓ **GUIDED PRACTICE**   for Example 1

Evaluate the expression.

1. $20 - 4^2$ **2.** $2 \cdot 3^2 + 4$ **3.** $32 \div 2^3 + 6$ **4.** $15 + 6^2 - 4$

GROUPING SYMBOLS Grouping symbols such as parentheses ( ) and brackets [ ] indicate that operations inside the grouping symbols should be performed first. For example, to evaluate 2 • 4 + 6, you multiply first, then add. To evaluate 2(4 + 6), you add first, then multiply.

EXAMPLE 2 **Evaluate expressions with grouping symbols**

Evaluate the expression.

a. $7(\mathbf{13 - 8}) = 7(\mathbf{5})$ **Subtract within parentheses.**

$= 35$ **Multiply.**

b. $24 - (\mathbf{3^2} + 1) = 24 - (\mathbf{9} + 1)$ **Evaluate power.**

$= 24 - 10$ **Add within parentheses.**

$= 14$ **Subtract.**

AVOID ERRORS
When grouping symbols appear inside other grouping symbols, work from the innermost grouping symbols outward.

c. $2[30 - (\mathbf{8 + 13})] = 2[30 - \mathbf{21}]$ **Add within parentheses.**

$= 2[9]$ **Subtract within brackets.**

$= 18$ **Multiply.**

FRACTION BARS A fraction bar can act as a grouping symbol. Evaluate the numerator and denominator before you divide:

$$\frac{8 + 4}{5 - 2} = (8 + 4) \div (5 - 2) = 12 \div 3 = 4$$

EXAMPLE 3 **Evaluate an algebraic expression**

Evaluate the expression when $x = 4$.

$$\frac{9x}{3(x + 2)} = \frac{9 \cdot \mathbf{4}}{3(\mathbf{4} + 2)}$$ **Substitute 4 for x.**

$$= \frac{9 \cdot 4}{3 \cdot 6}$$ **Add within parentheses.**

$$= \frac{36}{18}$$ **Multiply.**

$$= 2$$ **Divide.**

Animated **Algebra** at my.hrw.com

✓ **GUIDED PRACTICE**   for Examples 2 and 3

Evaluate the expression.

5. $4(3 + 9)$ **6.** $3(8 - 2^2)$ **7.** $2[(9 + 3) \div 4]$

Evaluate the expression when $y = 8$.

8. $y^2 - 3$ **9.** $12 - y - 1$ **10.** $\dfrac{10y + 1}{y + 1}$

★ **EXAMPLE 4**   **Standardized Test Practice**

A group of 12 students volunteers to collect litter for one day. A sponsor provides 3 juice drinks and 2 sandwiches for each student and pays $30 for trash bags. The sponsor's cost (in dollars) is given by the expression $12(3j + 2s) + 30$ where j is the cost of a juice drink and s is the cost of a sandwich. A juice drink costs $1.25. A sandwich costs $2. What is the sponsor's cost?

(A) $79   (B) $123   (C) $129   (D) $210

Solution

$$12(3j + 2s) + 30 = 12(3 \cdot 1.25 + 2 \cdot 2) + 30 \qquad \text{Substitute 1.25 for } j \text{ and 2 for } s.$$
$$= 12(3.75 + 4) + 30 \qquad \text{Multiply within parentheses.}$$
$$= 12(7.75) + 30 \qquad \text{Add within parentheses.}$$
$$= 93 + 30 \qquad \text{Multiply.}$$
$$= 123 \qquad \text{Add.}$$

▶ The sponsor's cost is $123. The correct answer is B.   (A) (B) (C) (D).

✓ **GUIDED PRACTICE**   for Example 4

11. WHAT IF? In Example 4, suppose the number of volunteers doubles. Does the sponsor's cost double as well? *Explain*.

1.2 EXERCISES

SKILL PRACTICE

1. VOCABULARY According to the order of operations, which operation would you perform first in simplifying $50 - 5 \times 4^2 \div 2$?

2. ★ WRITING *Describe* the steps you would use to evaluate the expression $2(3x + 1)^2$ when $x = 3$.

EXAMPLES 1 and 2
for Exs. 3–21

EVALUATING EXPRESSIONS Evaluate the expression.

3. $13 - 8 + 3$

4. $8 - 2^2$

5. $3 \cdot 6 - 4$

6. $5 \cdot 2^3 + 7$

7. $48 \div 4^2 + \dfrac{3}{5}$

8. $1 + 5^2 \div 50$

9. $2^4 \cdot 4 - 2 \div 8$

10. $4^3 \div 8 + 8$

11. $(12 + 72) \div 4$

12. $24 + 4(3 + 1)$

13. $12(6 - 3.5)^2 - 1.5$

14. $24 \div (8 + 4^2)$

15. $\dfrac{1}{2}(21 + 2^2)$

16. $\dfrac{1}{6}(6 + 18) - 2^2$

17. $\dfrac{3}{4}[13 - (2 + 3)]^2$

18. $8[20 - (9 - 5)^2]$

19. ★ **MULTIPLE CHOICE** What is the value of $3[20 - (7 - 5)^2]$?

    A 48   **B** 56   **C** 192   **D** 972

ERROR ANALYSIS *Describe* and correct the error in evaluating the expression.

20.
$$(1 + 13) \div 7 + 7 = 14 \div 7 + 7$$
$$= 14 \div 14$$
$$= 1$$
✗

21.
$$20 - \frac{1}{2} \cdot 6^2 = 20 - 3^2$$
$$= 20 - 9$$
$$= 11$$
✗

EXAMPLE 3
for Exs. 22–31

EVALUATING EXPRESSIONS Evaluate the expression.

22. $4n - 12$ when $n = 7$ **23.** $2 + 3x^2$ when $x = 3$ **24.** $6t^2 - 13$ when $t = 2$

25. $11 + r^3 - 2r$ when $r = 5$ **26.** $5(w - 4)$ when $w = 7$ **27.** $3(m^2 - 2)$ when $m = 1.5$

28. $\frac{9x + 4}{3x + 1}$ when $x = 7$ **29.** $\frac{k^2 - 1}{k + 3}$ when $k = 5$ **30.** $\frac{b^3 - 21}{5b + 9}$ when $b = 3$

31. ★ **MULTIPLE CHOICE** What is the value of $\frac{x^2}{25} + 3x$ when $x = 10$?

    A 26   **B** 34   **C** 43   **D** 105

CHALLENGE Insert grouping symbols in the expression so that the value of the expression is 14.

32. $9 + 39 + 22 \div 11 - 9 + 3$ **33.** $2 \times 2 + 3^2 - 4 + 3 \times 5$

PROBLEM SOLVING

EXAMPLE 4
for Exs. 34–37

34. SALES Your school's booster club sells school T-shirts. Half the T-shirts come from one supplier at a cost of $5.95 each, and half from another supplier at a cost of $6.15 each. The average cost (in dollars) of a T-shirt is given by the expression $\frac{5.95 + 6.15}{2}$. Find the average cost.

35. **MULTI-STEP PROBLEM** You join an online music service. The total cost (in dollars) of downloading 3 singles at $.99 each and 2 albums at $9.95 each is given by the expression $3 \cdot 0.99 + 2 \cdot 9.95$.

  a. Find the total cost.

  b. You have $25 to spend. How much will you have left?

36. PHYSIOLOGY If you know how tall you were at the age of 2, you can estimate your adult height (in inches). Girls can use the expression $25 + 1.17h$ where h is the height (in inches) at the age of 2. Boys can use the expression $22.7 + 1.37h$. Estimate the adult height of each person to the nearest inch.

  a. A girl who was 34 inches tall at age 2

  b. A boy who was 33 inches tall at age 2

37. ★ **OPEN-ENDED** Write a numerical expression including parentheses that has the same value when you remove the parentheses.

38. **ONLINE SHOPPING** The regular shipping fee (in dollars) for an online computer store is given by the expression $0.5w + 4.49$ where w is the weight (in pounds) of the item. The fee (in dollars) for rush delivery is given by $0.99w + 6.49$. You purchase a 26.5 pound computer. How much do you save using regular shipping instead of rush delivery?

39. ★ **SHORT RESPONSE** You make and sell flags for $10 each. Each flag requires $4.50 worth of fabric. You pay $12.99 for a kit to punch holes to hang the flags. Your expenses (in dollars) are given by the expression $4.50m + 12.99$ where m is the number of flags you make. Your income is given by the expression $10s$ where s is the number of flags you sell. Your profit is equal to the difference of your income and your expenses.

   a. You make 50 flags and sell 38 of them. Find your income and your expenses. Then find your profit.

   b. *Explain* how you could use a single expression to determine your profit.

40. ★ **EXTENDED RESPONSE** Each year Heisman Trophy voters select the outstanding college football player. Each voter selects three players ranked first to third. A first place vote is worth 3 points, a second place vote is worth 2 points, and a third place vote is worth 1 point. Let f, s, and t be, respectively, the number of first place, second place, and third place votes a player gets. The table shows the votes for the winner and the runner-up in 2003.

Player	First place	Second place	Third place
Jason White	319	204	116
Larry Fitzgerald	253	233	128

   a. **Analyze** *Explain* why the expression $3f + 2s + t$ represents a player's point total.

   b. **Calculate** Use the expression in part (a) to determine how many more points Jason White got than Larry Fitzgerald got.

   c. **CHALLENGE** Can you rearrange the order of the votes for each player in such a way that Larry Fitzgerald would have won? *Explain*.

Use Order of Operations

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

QUESTION How can you use a graphing calculator to evaluate an expression?

You can use a graphing calculator to evaluate an expression. When you enter the expression, it is important to use grouping symbols so that the calculator performs operations in the correct order.

EXAMPLE Evaluate an expression

Use a graphing calculator to evaluate an expression.

Lean body mass is the mass of the skeleton, muscles, and organs. Physicians use lean body mass to determine dosages of medicine.

Scientists have developed separate formulas for the lean body masses of men and women based on their mass m (in kilograms) and height h (in meters). Lean body mass in measured in units called BMI (Body Mass Index) units.

Men: $1.10m - \dfrac{128m^2}{10{,}000h^2}$ **Women:** $1.07m - \dfrac{148m^2}{10{,}000h^2}$

Find the lean body mass (in BMI units) of a man who is 1.8 meters tall and has a mass of 80 kilograms.
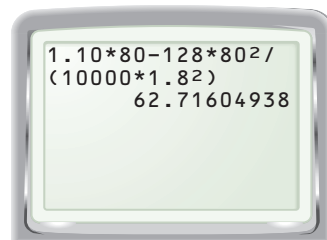
Solution

Enter the expression for men in the calculator. Substitute 80 for m and 1.8 for h. Because the fraction bar is a grouping symbol, enter the denominator using parentheses.

Use the following keystrokes.

1.10 ⊠ 80 ⊟ 128 ⊠ 80 x^2 ⊡ (10000 ⊠ 1.8 x^2)

```
1.10*80-128*80²/
(10000*1.8²)
        62.71604938
```

▶ The lean body mass of a man who is 1.8 meters tall and has a mass of 80 kilograms is about 62.7 BMI units.

PRACTICE

Use a calculator to evaluate the expression for $n = 4$. Round to the nearest thousandth.

1. $3 + 5 \cdot n \div 10$ **2.** $2 + \dfrac{3n^2}{4}$ **3.** $\dfrac{83}{3n^2} - 1.3$

4. $\dfrac{14.2n}{8 + n^3}$ **5.** $\dfrac{7 - n}{n^2}$ **6.** $5n^2 + \dfrac{4n^3 + 1}{3}$

7. Find the lean body mass (to the nearest tenth of a BMI unit) of a woman who is 1.6 meters tall and has a mass of 54 kilograms.

Patterns and Expressions
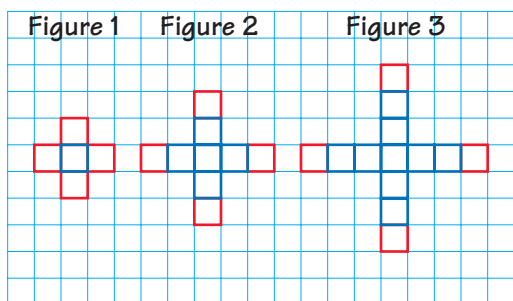
MATERIALS • graph paper

MATHEMATICAL PRACTICES

Look for and make use of structure.

QUESTION How can you use an algebraic expression to describe a pattern?

EXPLORE Create and describe a pattern

STEP 1

Figure 1   Figure 2   Figure 3

STEP 2

Figure 1   Figure 2   Figure 3

1 + 4   1 + 4 + 4   1 + 4 + 4 + 4

Draw a figure Draw a unit square on graph paper. Then draw a unit square against each side of the first square to form figure 1.

Copy figure 1 and draw a square on each "arm" to form figure 2. Use the same method to form figure 3.
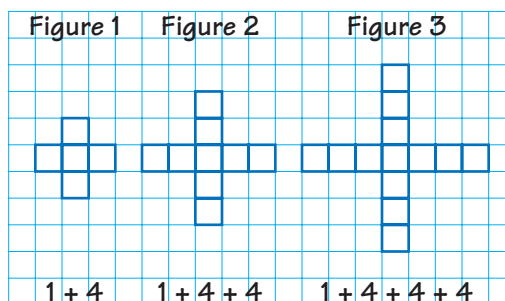
Write expressions For each figure, write a numerical expression that describes the number of squares in the figure.

DRAW CONCLUSIONS Use your observations to complete these exercises

In Exercises 1–3, use the pattern in Steps 1 and 2 above.

1. How is the figure number related to the number of times 4 is added in the numerical expression? Predict the number of squares in the fourth figure. Create figure 4 and check your prediction.

2. *Describe* how to calculate the number of squares in the nth figure.

3. Write an algebraic expression for the number of squares in the nth figure. (*Hint:* Remember that repeated addition can be written as multiplication.)

4. **a.** Write an algebraic expression for the number of squares in the nth figure of the pattern shown.

   b. *Explain* why the expression n^2 is not an appropriate answer to part (a). Create a pattern that can be described by the expression n^2.

Figure 1   **Figure 2**   **Figure 3**   **Figure 4**