# 1.7 Represent Functions as Rules and Tables

**Before** You wrote algebraic expressions and equations.

**Now** You will represent functions as rules and as tables.

**Why?** So you can describe consumer costs, as in Example 1.

**Key Vocabulary**
- **function**
- **domain**
- **range**
- **independent variable**
- **dependent variable**

When you pump gas, the total cost depends on the number of gallons pumped. The total cost is a *function* of the number of gallons pumped.

A **function** consists of:

- A set called the **domain** containing numbers called **inputs**, and a set called the **range** containing numbers called **outputs**.
- A pairing of inputs with outputs such that each input is paired with exactly one output.

COMMON CORE

**CC.9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

## EXAMPLE 1 Identify the domain and range of a function

The input-output table shows the cost of various amounts of regular unleaded gas from the same pump. Identify the domain and range of the function.


| Input (gallons) | 10 | 12 | 13 | 17 |
|---|---|---|---|---|
| Output (dollars) | 19.99 | 23.99 | 25.99 | 33.98 |

### Solution

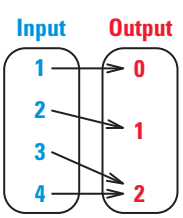
▶ The domain is the set of inputs: 10, 12, 13, and 17. The range is the set of outputs: 19.99, 23.99, 25.99, and 33.98.

## ✓ GUIDED PRACTICE for Example 1

**1.** Identify the domain and range of the function.

| Input | 0 | 1 | 2 | 4 |
|---|---|---|---|---|
| Output | 5 | 2 | 2 | 1 |

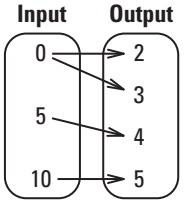
**MAPPING DIAGRAMS** A function may be represented by a *mapping diagram*. Notice that an output may be paired with more than one input, but no input is paired with more than one output.



## EXAMPLE 2 Identify a function

**Tell whether the pairing is a function.**

**a.**




The pairing is *not* a function because the input 0 is paired with both 2 and 3.

**b.**

| Input | Output |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 4 | 8 |
| 6 | 12 |

The pairing is a function because each input is paired with exactly one output.

## GUIDED PRACTICE for Example 2

**Tell whether the pairing is a function.**

**2.**

| Input | 3 | 6 | 9 | 12 |
|---|---|---|---|---|
| Output | 1 | 2 | 2 | 1 |

**3.**

| Input | 2 | 2 | 4 | 7 |
|---|---|---|---|---|
| Output | 0 | 1 | 2 | 3 |

**FUNCTION RULES** A function may be represented using a rule that relates one variable to another. The input variable is called the **independent variable.** The output variable is called the **dependent variable** because its value depends on the value of the input variable.

**READING**
Function rules typically give the dependent variable in terms of the independent variable. In an equation like $y = x + 3$, you know that $y$ is the dependent variable.

### KEY CONCEPT *For Your Notebook*

### Functions

**Verbal Rule**

The output is 3 more than the input.

**Equation**

$y = x + 3$

**Table**

| Input, $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Output, $y$ | 3 | 4 | 5 | 6 | 7 |

## EXAMPLE 3 Make a table for a function

**The domain of the function $y = 2x$ is 0, 2, 5, 7, and 8. Make a table for the function, then identify the range of the function.**

**Solution**

| $x$ | 0 | 2 | 5 | 7 | 8 |
|---|---|---|---|---|---|
| $y = 2x$ | $2(0) = 0$ | $2(2) = 4$ | $2(5) = 10$ | $2(7) = 14$ | $2(8) = 16$ |

The range of the function is 0, 4, 10, 14, and 16.

## EXAMPLE 4 Write a function rule

**Write a rule for the function.**

| Input | 0 | 1 | 4 | 6 | 10 |
|---|---|---|---|---|---|
| Output | 2 | 3 | 6 | 8 | 12 |

**Solution**

Let $x$ be the input, or independent variable, and let $y$ be the output, or dependent variable. Notice that each output is 2 more than the corresponding input. So, a rule for the function is $y = x + 2$.

## EXAMPLE 5 Write a function rule for a real-world situation

**CONCERT TICKETS** You are buying concert tickets that cost \$15 each. You can buy up to 6 tickets. Write the amount (in dollars) you spend as a function of the number of tickets you buy. Identify the independent and dependent variables. Then identify the domain and the range of the function.

**Solution**

**CHOOSE A VARIABLE**
To write a function rule for a real-world situation, choose letters for the variables that remind you of the quantities represented.

Write a verbal model. Then write a function rule. Let $n$ represent the number of tickets purchased and $A$ represent the amount spent (in dollars).

| Amount spent (dollars) | = | Cost per ticket (dollars/ticket) | · | Tickets purchased (tickets) |
|---|---|---|---|---|
| ↓ | | ↓ | | ↓ |
| $A$ | = | 15 | · | $n$ |

So, the function rule is $A = 15n$. The amount spent depends on the number of tickets bought, so $n$ is the independent variable and $A$ is the dependent variable.

Because you can buy up to 6 tickets, the domain of the function is 0, 1, 2, 3, 4, 5, and 6. Make a table to identify the range.

| Number of tickets, $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Amount (dollars), $A$ | 0 | 15 | 30 | 45 | 60 | 75 | 90 |

The range of the function is 0, 15, 30, 45, 60, 75, and 90.

Animated Algebra at my.hrw.com

## GUIDED PRACTICE for Examples 3, 4, and 5

**4.** Make a table for the function $y = x - 5$ with domain 10, 12, 15, 18, and 29. Then identify the range of the function.

**5.** Write a rule for the function. Identify the domain and the range.

| Time (hours) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Pay (dollars) | 8 | 16 | 24 | 32 |

# 1.7 EXERCISES

**HOMEWORK KEY**

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 7 and 23
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 12, 13, 26, and 27
- ◆ = **MULTIPLE REPRESENTATIONS** Exs. 23 and 24

## SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A(n) ? is a number in the domain of a function. A(n) ? is a number in the range of a function.

2. ★ **WRITING** In the equation $b = a - 2$, which variable is the independent variable and which is the dependent variable? *Explain.*

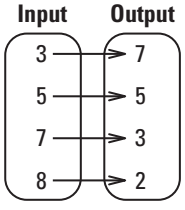
**EXAMPLES 1 and 2** for Exs. 3–11

**DOMAIN AND RANGE** **Identify the domain and range of the function.**

3.

| Input | Output |
|---|---|
| 0 | 5 |
| 1 | 7 |
| 2 | 15 |
| 3 | 44 |

4.




5.

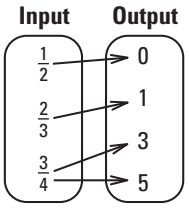
| Input | Output |
|---|---|
| 6 | 5 |
| 12 | 7 |
| 21 | 10 |
| 42 | 17 |

**IDENTIFYING FUNCTIONS** **Tell whether the pairing is a function.**

6.

| Input | Output |
|---|---|
| 0 | 7.5 |
| 1 | 9.5 |
| 2 | 11.5 |
| 3 | 13.5 |

7.




8.

| Input | Output |
|---|---|
| 7 | 13 |
| 11 | 8 |
| 21 | 13 |
| 35 | 20 |

**ERROR ANALYSIS** **In Exercises 9 and 10, describe and correct the error related to the function represented by the table.**

| Input, $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Output, $y$ | 6 | 7 | 8 | 6 | 9 |

9. The pairing is not a function. One output is paired with two inputs. ✗

10. The pairing is a function. The range is 1, 2, 3, 4, and 5. ✗

11. ★ **OPEN-ENDED** Draw a mapping diagram for a function with 6 inputs. Then make a table to represent the function.

**EXAMPLES 3 and 4** for Exs. 12–21

12. ★ **MULTIPLE CHOICE** The domain of the function $y = 5x - 1$ is 1, 3, 4, 5, and 6. Which number is in the range of the function?

(A) 0 (B) 4 (C) 9 (D) 15

13. ★ **MULTIPLE CHOICE** Each output of a function is 0.5 less than the corresponding input. Which equation is a rule for the function?

(A) $y = x - 0.5$ (B) $y = x + 0.5$ (C) $y = 0.5 - x$ (D) $y = 0.5x$

**TABLES** **Make a table for the function. Identify the range of the function.**

**14.** $y = x - 3$
Domain: 12, 15, 22, 30

**15.** $y = x + 3.5$
Domain: 4, 5, 7, 8, 12

**16.** $y = 3x + 4$
Domain: 0, 5, 7, 10

**17.** $y = \frac{1}{2}x + 3$
Domain: 4, 6, 9, 11

**18.** $y = \frac{2}{3}x + \frac{1}{3}$
Domain: 4, 6, 8, 12

**19.** $y = \frac{0.5x + 1}{2}$
Domain: 0, 2, 4, 6

**FUNCTION RULES** **Write a rule for the function.**

**20.**

| Input, $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Output, $y$ | 2.2 | 3.2 | 4.2 | 5.2 |

**21.**

| Input, $x$ | 15 | 20 | 21 | 30 | 42 |
|---|---|---|---|---|---|
| Output, $y$ | 7 | 12 | 13 | 22 | 34 |

**22.** **CHALLENGE** Fill in the table in such a way that when $t$ is the independent variable, the pairing is a function, and when $t$ is the dependent variable, the pairing is not a function.

| $t$ | ? | ? | ? | ? |
|---|---|---|---|---|
| $v$ | ? | ? | ? | ? |

## PROBLEM SOLVING

**EXAMPLE 5** for Exs. 23–26

**23.** **MULTIPLE REPRESENTATIONS** You have 10 quarters that you can use for a parking meter.

**a. Describing in Words** Copy and complete: Each time you put 1 quarter in the meter, you have 1 less quarter, so ? is a function of ?.

**b. Writing a Rule** Write a rule for the number $y$ of quarters that you have left as a function of the number $x$ of quarters you have used so far. Identify the domain of the function.

**c. Making a Table** Make a table and identify the range of the function.

**24.** **MULTIPLE REPRESENTATIONS** At a yard sale, you find 5 paperback books by your favorite author. Each book is priced at $.75.

**a. Describing in Words** Copy and complete: For each book you buy, you spend $.75, so ? is a function of ?.

**b. Writing a Rule** Write a rule for the amount (in dollars) you spend as a function of the number of books you buy. Identify the domain of the function.

**c. Making a Table** Make a table and identify the range of the function.

**25.** **SAVINGS** You have $100 saved and plan to save $20 each month. Write a rule for the amount saved (in dollars) as a function of the number of months from now. Identify the independent and dependent variables, the domain, and the range. How much will you have saved altogether 12 months from now?

**26.** ★ **OPEN-ENDED** Write a function rule that models a real-world situation. Identify the independent variable and the dependent variable.

**27.** ★ **SHORT RESPONSE** Consider a pairing of the digits 2 through 9 on a telephone keypad with the associated letters.


**a.** Make a table showing the pairing with the digits as inputs and the letters as outputs. Is the pairing a function? *Explain.*

**b.** Make a table showing the pairing with the letters as inputs and the digits as outputs. Is the pairing a function? *Explain.*

**28.** **MULTI-STEP PROBLEM** The table shows the fuel efficiency of four compact cars from one manufacturer for model year 2004.

| City fuel efficiency (mi/gal), $c$ | 24 | 26 | 27 | 28 |
|---|---|---|---|---|
| Highway fuel efficiency (mi/gal), $h$ | 32 | 34 | 35 | 36 |

**a.** **Write a Rule** Use the table to write a rule for the cars' highway fuel efficiency as a function of their city fuel efficiency.

**b.** **Predict** Another of the manufacturer's compact cars has a city fuel efficiency of 30 miles per gallon. Predict the highway fuel efficiency.

**c.** **Calculate** A study found that if gas costs \$2 per gallon, you can use the expression $\frac{11{,}550}{c} + \frac{9450}{h}$ to estimate a car's annual fuel cost (in dollars) for a typical driver. Evaluate the expression for the car in part (b).

**29.** **CHALLENGE** Each week you spend a total of 5 hours exercising. You swim part of the time and bike the rest.

300 calories per hour

440 calories per hour

**a.** Write a rule for the total number of calories you burn for the whole 5 hours as a function of the time you spend swimming.

**b.** One week you spend half the time swimming. How many calories do you burn during the whole 5 hours?

my.hrw.com
Keystrokes

# Make a Table

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

## QUESTION How can you use a graphing calculator to create a table for a function?

You can use a graphing calculator to create a table for a function when you want to display many pairs of input values and output values or when you want to find the input value that corresponds to a given output value.

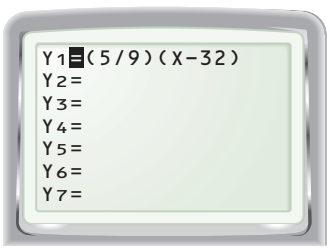
In the example below, you will make a table to compare temperatures in degrees Celsius and temperatures in degrees Fahrenheit for temperatures at or above the temperature at which water freezes, 32°F.

## EXAMPLE Use a graphing calculator to make a table

The formula $C = \frac{5}{9}(F - 32)$ gives the temperature in degrees Celsius as a function of the temperature in degrees Fahrenheit. Make a table for the function.

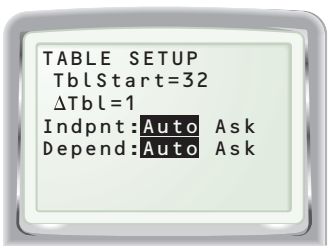
### STEP 1 Enter equation

Rewrite the function using $x$ for $F$ and $y$ for $C$. Press Y= and enter $\frac{5}{9}(x - 32)$.



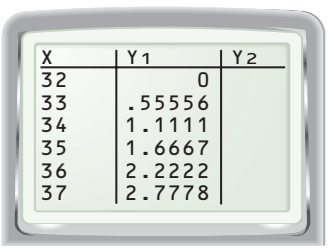

### STEP 2 Set up table

Go to the TABLE SETUP screen. Use a starting value (TblStart) of 32 and an increment (△Tbl) of 1.




### STEP 3 View table

Display the table. Scroll down to see pairs of inputs and outputs.




## PRACTICE

1. You see a sign that indicates that the outdoor temperature is 10°C. Find the temperature in degrees Fahrenheit. *Explain* how you found your answer.
2. Water boils at 100°C. What is the temperature in degrees Fahrenheit?

**Make a table for the function. Use the given starting value and increment.**

3. $y = \frac{3}{4}x + 5$
   TblStart = 0, △Tbl = 1
4. $y = 4x + 2$
   TblStart = 0, △Tbl = 0.5
5. $y = 7.5x - 0.5$
   TblStart = 1, △Tbl = 1
6. $y = 0.5x + 6$
   TblStart = 3, △Tbl = 3

# Scatter Plots and Functions

**MATERIALS** • tape measure • graph paper

MATHEMATICAL PRACTICES

**Model with mathematics.**

## QUESTION How can you tell whether a graph represents a function?

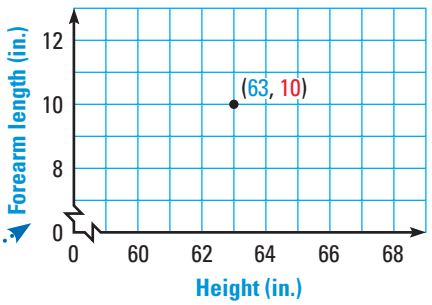
A *scatter plot* is a type of display for paired data. Each data pair is plotted as a point. In this activity, you will work in a group to make a scatter plot. You will measure the height of each student in your group and the length of his or her forearm. The length of the forearm is the distance from the elbow to the wrist.

## EXPLORE Collect data and make a scatter plot

**STEP 1** ***Collect data*** Measure the height of each student in your group and the length of his or her forearm. Record the results for each student in one row of a table like the one shown.

| Height (inches) | Forearm length (inches) |
|---|---|
| 63 | 10 |
| ? | ? |

**STEP 2** ***Make a scatter plot*** Use graph paper to draw axes labeled as shown. Then plot the data pairs (*height*, *forearm length*). For example, plot the point (**63**, **10**) for a student with a height of 63 inches and a forearm length of 10 inches.




The symbol ⌇ on an axis represents a break in the axis.

## DRAW CONCLUSIONS Use your observations to complete these exercises

1. Examine your scatter plot. What does it suggest about the relationship between a person's height and the person's forearm length?
2. Compare your table with those of the other groups in your class. Determine which of the tables represent functions and which do not.
3. Is it possible to determine whether a table represents a function by looking at the corresponding scatter plot? *Explain*.