Name ______________________________ Date __________ Class ___________

SKILL 89

Skills Readiness

Counterexamples

Definition: A counterexample to a statement is a particular example or instance of the statement that is NOT true.

> Example: Find a counterexample to show that the statement below is false.
>
> Statement: $1 + n \geq 1 - n$, where n is a real number.
>
> Strategy to find a counterexample:
>
> Step 1: Always try $n = 0$ first. $1 + 0 \geq 1 - 0 \rightarrow 1 \geq 1$ **True**
>
> Step 2: Try $n = 1$ next. $1 + 1 \geq 1 - 1 \rightarrow 2 \geq 0$ **True**
>
> Step 3: Try $n = -1$ next. $1 + (-1) \geq 1 - (-1) \rightarrow 0 \geq 2$ **False**
>
> **STOP: $n = -1$ is your counterexample.**
>
> If $n = -1$ had produced another true statement, then you would try a positive and negative fraction, such as $\frac{1}{2}$ and $-\frac{1}{2}$. Then larger numbers, both positive and negative, until you found a number that produced a false statement.

Practice on Your Own

Find a counterexample to show that each statement is false.

1. $n^3 + 2n = 3n^2$, where n is a real number

$n =$

2. $-\frac{1}{n} \leq \frac{1}{n}$, where n is a real number

$n =$

3. $\frac{n}{3} \neq \frac{3}{n}$, where n is a real number

$n =$

4. $\frac{n}{2} < 2n$, where n is a real number

$n =$

5. $-(n + 1) \neq n + 1$, where n is a real number

$n =$

6. $n^2 \geq n$, where n is a real number

$n =$

Check

Find a counterexample to show that each statement is false.

7. $\frac{1}{n} \neq \frac{1}{n^2}$, where n is a real number $\neq 0$

$n =$

8. $n^3 \geq n^2$, where n is a real number

$n =$

9. $\frac{n}{5} < 5n$, where n is a real number

$n =$

10. $-3n \neq 3n$, where n is a real number

$n =$