

Study Guide and Intervention

Solving Quadratic Equations by Completing the Square

Find the Square Root An equation such as $x^2 - 4x + 4 = 5$ can be solved by taking the square root of each side.

Example 1 Solve $x^2 - 2x + 1 = 9$.

Round to the nearest tenth if necessary.

$$x^2 - 2x + 1 = 9$$

$$(x - 1)^2 = 9$$

$$\sqrt{(x - 1)^2} = \sqrt{9}$$

$$|x - 1| = \sqrt{9}$$

$$x - 1 = \pm 3$$

$$x - 1 + 1 = \pm 3 + 1$$

$$x = 1 \pm 3$$

$$x = 1 + 3 \quad \text{or} \quad x = 1 - 3$$

$$= 4 \quad \quad \quad = -2$$

The solution set is $\{-2, 4\}$.

Example 2 Solve $x^2 - 4x + 4 = 5$.

Round to the nearest tenth if necessary.

$$x^2 - 4x + 4 = 5$$

$$(x - 2)^2 = 5$$

$$\sqrt{(x - 2)^2} = \sqrt{5}$$

$$|x - 2| = \sqrt{5}$$

$$x - 2 = \pm\sqrt{5}$$

$$x - 2 + 2 = \pm\sqrt{5} + 2$$

$$x = 2 \pm \sqrt{5}$$

Use a calculator to evaluate each value of x .

$$x = 2 + \sqrt{5} \quad \text{or} \quad x = 2 - \sqrt{5}$$

$$\approx 4.2 \quad \quad \quad \approx -0.2$$

The solution set is $\{-0.2, 4.2\}$.

Exercises

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $x^2 + 4x + 4 = 9$

2. $m^2 + 12m + 36 = 1$

3. $r^2 - 6r + 9 = 16$

4. $x^2 - 2x + 1 = 25$

5. $x^2 - 8x + 16 = 5$

6. $x^2 - 10x + 25 = 8$

7. $c^2 - 4c + 4 = 7$

8. $p^2 + 16p + 64 = 3$

9. $x^2 + 8x + 16 = 9$

10. $x^2 + 6x + 9 = 4$

11. $a^2 + 8a + 16 = 10$

12. $y^2 - 12y + 36 = 5$

13. $x^2 + 10x + 25 = 1$

14. $y^2 + 14y + 49 = 6$

15. $m^2 - 8m + 16 = 2$

16. $x^2 + 12x + 36 = 10$

17. $a^2 - 14a + 49 = 3$

18. $y^2 + 8y + 16 = 7$

Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Completing the Square

Complete the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

Step 1	Find $\frac{b}{2}$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$.

Example

Solve $x^2 + 6x + 3 = 10$ by completing the square.

$$\begin{array}{ll}
 x^2 + 6x + 3 = 10 & \text{Original equation} \\
 x^2 + 6x + 3 - 3 = 10 - 3 & \text{Subtract 3 from each side.} \\
 x^2 + 6x = 7 & \text{Simplify.} \\
 x^2 + 6x + 9 = 7 + 9 & \text{Since } \left(\frac{6}{2}\right)^2 = 9, \text{ add 9 to each side.} \\
 (x + 3)^2 = 16 & \text{Factor } x^2 + 6x + 9. \\
 x + 3 = \pm 4 & \text{Take the square root of each side.} \\
 x = -3 \pm 4 & \text{Simplify.} \\
 x = -3 + 4 \quad \text{or} \quad x = -3 - 4 & \\
 = 1 & \qquad \qquad = -7
 \end{array}$$

The solution set is $\{-7, 1\}$.

Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

1. $t^2 - 4t + 3 = 0$

2. $y^2 + 10y = -9$

3. $y^2 - 8y - 9 = 0$

4. $x^2 - 6x = 16$

5. $p^2 - 4p - 5 = 0$

6. $x^2 - 12x = 9$

7. $c^2 + 8c = 20$

8. $p^2 = 2p + 1$

9. $x^2 + 20x + 11 = -8$

10. $x^2 - 1 = 5x$

11. $a^2 = 22a + 23$

12. $m^2 - 8m = -7$

13. $x^2 + 10x = 24$

14. $a^2 - 18a = 19$

15. $b^2 + 16b = -16$

16. $4x^2 = 24 + 4x$

17. $2m^2 + 4m + 2 = 8$

18. $4k^2 = 40k + 44$