

Study Guide and Intervention

Factors and Greatest Common Factors

Prime Factorization When two or more numbers are multiplied, each number is called a **factor** of the product.

	Definition	Example
Prime Number	A prime number is a whole number, greater than 1, whose only factors are 1 and itself.	5
Composite Number	A composite number is a whole number, greater than 1, that has more than two factors.	10
Prime Factorization	Prime factorization occurs when a whole number is expressed as a product of factors that are all prime numbers.	$45 = 3^2 \cdot 5$

Example 1 Factor each number.

Then classify each number as *prime* or *composite*.

a. 28

To find the factors of 28, list all pairs of whole numbers whose product is 28.

$$1 \times 28 \quad 2 \times 14 \quad 4 \times 7$$

Therefore, the factors of 28 are 1, 2, 4, 7, 14, and 28. Since 28 has more than 2 factors, it is a composite number.

b. 31

To find the factors of 31, list all pairs of whole numbers whose product is 31.

$$1 \times 31$$

Therefore, the factors of 31 are 1 and 31. Since the only factors of 31 are itself and 1, it is a prime number.

Example 2 Find the prime factorization of 200.

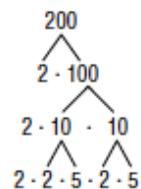
Method 1

$$\begin{aligned} 200 &= 2 \cdot 100 \\ &= 2 \cdot 2 \cdot 50 \\ &= 2 \cdot 2 \cdot 2 \cdot 25 \\ &= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \end{aligned}$$

All the factors in the last row are prime, so the prime factorization of 200 is $2^3 \cdot 5^2$.

Method 2

Use a factor tree.



All of the factors in each last branch of the factor tree are prime, so the prime factorization of 200 is $2^3 \cdot 5^2$.

Exercises

Find the factors of each number. Then classify the number as *prime* or *composite*.

1. 41

2. 121

3. 90

4. 2865

Find the prime factorization of each integer.

5. 600

6. 175

7. -150

Factor each monomial completely.

8. $32x^2$

9. $18m^2n$

10. $49a^3b^2$

Study Guide and Intervention *(continued)*

Factors and Greatest Common Factors

Greatest Common Factor

Greatest Common Factor (GCF)	
Integers	the greatest number that is a factor of all the integers
Monomials	the product of their common factors when each monomial is expressed in factored form

If two or more integers or monomials have no common prime factors, their GCF is 1 and the integers or monomials are said to be **relatively prime**.

Example

Find the GCF of each set of monomials.

a. 12 and 18

$$12 = \textcircled{2} \cdot 2 \cdot \textcircled{3} \quad \text{Factor each number.}$$

$$18 = \textcircled{2} \cdot \textcircled{3} \cdot 3 \quad \text{Circle the common prime factors, if any.}$$

The GCF of 12 and 18 is $2 \cdot 3$ or 6.

b. $16xy^2z^2$ and $72xyz^3$

$$16xy^2z^2 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{2} \cdot 2 \cdot \textcircled{x} \cdot \textcircled{y} \cdot y \cdot \textcircled{z} \cdot \textcircled{z}$$

$$72xyz^3 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{2} \cdot 3 \cdot 3 \cdot \textcircled{x} \cdot \textcircled{y} \cdot \textcircled{z} \cdot \textcircled{z} \cdot z$$

The GCF of $16xy^2z^2$ and $72xyz^3$ is $2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot z \cdot z$ or $8xyz^2$.

Exercises

Find the GCF of each set of monomials.

- | | | |
|------------------------------------|----------------------------------|-------------------------------------|
| 1. 12, 48 | 2. 18, 42 | 3. 64, 80 |
| 4. 32, 54 | 5. 27, 32 | 6. 44, 100 |
| 7. 45, 15 | 8. 169, 13 | 9. 20, 440 |
| 10. $49x$, $343x^2$ | 11. $4a^7b$, $28ab$ | 12. $96y$, $12x$, $8y$ |
| 13. $12a$, $18abc$ | 14. $28y^2$, $35xy$, $49x^2yz$ | 15. $2m^2n$, $12mn^2$, $18mn$ |
| 16. $12x^2$, $32x^2yz$, $60xy^2$ | 17. $18a^3b^2$, $36a^3b^2$ | 18. $15mn^2$, $30m^3n^2$, $90m^3$ |
| 19. $2x^2y$, $9x^2y^3$, $18xy^2$ | 20. a^4b , $8a^3b^2$ | 21. ab^2 , $5a^4b^2$, $10b^3$ |