

Study Guide and Intervention

Solving Quadratic Equations by Using the Quadratic Formula

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the solutions of $ax^2 + bx + c = 0$, where $a \neq 0$
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Example 1 Solve $x^2 + 2x = 3$ by using the Quadratic Formula.

Rewrite the equation in standard form.

$$\begin{aligned} x^2 + 2x &= 3 && \text{Original equation} \\ x^2 + 2x - 3 &= 3 - 3 && \text{Subtract 3 from each side.} \\ x^2 + 2x - 3 &= 0 && \text{Simplify.} \end{aligned}$$

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{16}}{2} \end{aligned}$$

$$\begin{aligned} x &= \frac{-2 + 4}{2} \quad \text{or} \quad x = \frac{-2 - 4}{2} \\ &= 1 \quad \quad \quad = -3 \end{aligned}$$

The solution set is $\{-3, 1\}$.

Example 2 Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)} \\ &= \frac{6 \pm \sqrt{44}}{2} \\ x &= \frac{6 + \sqrt{44}}{2} \quad \text{or} \quad x = \frac{6 - \sqrt{44}}{2} \\ &\approx 6.3 \quad \quad \quad \approx -0.3 \end{aligned}$$

The solution set is $\{-0.3, 6.3\}$.

Exercises

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 3x + 2 = 0$

2. $m^2 - 8m = -16$

3. $16r^2 - 8r = -1$

4. $x^2 + 5x = 6$

5. $3x^2 + 2x = 8$

6. $8x^2 - 8x - 5 = 0$

7. $-4c^2 + 19c = 21$

8. $2p^2 + 6p = 5$

9. $48x^2 + 22x - 15 = 0$

10. $8x^2 - 4x = 24$

11. $2p^2 + 5p = 8$

12. $8y^2 + 9y - 4 = 0$

13. $2x^2 + 9x + 4 = 0$

14. $8y^2 + 17y + 2 = 0$

15. $3z^2 + 5z - 2 = 0$

16. $-2x^2 + 8x + 4 = 0$

17. $a^2 + 3a = 2$

18. $2y^2 - 6y + 4 = 0$

Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Using the Quadratic Formula

The Discriminant In the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real roots for a quadratic equation.

Case 1: $b^2 - 4ac < 0$	Case 2: $b^2 - 4ac = 0$	Case 3: $b^2 - 4ac > 0$
no real roots	one real root	two real roots

Example

State the value of the discriminant for each equation. Then determine the number of real roots.

a. $12x^2 + 5x = 4$

Write the equation in standard form.

$$12x^2 + 5x = 4 \quad \text{Original equation}$$

$$12x^2 + 5x - 4 = 4 - 4 \quad \text{Subtract 4 from each side.}$$

$$12x^2 + 5x - 4 = 0 \quad \text{Simplify.}$$

Now find the discriminant.

$$\begin{aligned} b^2 - 4ac &= (5)^2 - 4(12)(-4) \\ &= 217 \end{aligned}$$

Since the discriminant is positive, the equation has two real roots.

b. $2x^2 + 3x = -4$

$$2x^2 + 3x = -4 \quad \text{Original equation}$$

$$2x^2 + 3x + 4 = -4 + 4 \quad \text{Add 4 to each side.}$$

$$2x^2 + 3x + 4 = 0 \quad \text{Simplify.}$$

$$\begin{aligned} b^2 - 4ac &= (3)^2 - 4(2)(4) \\ &= -23 \end{aligned}$$

Since the discriminant is negative, the equation has no real roots.

Exercises

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

1. $3x^2 + 2x - 3 = 0$

2. $3n^2 - 7n - 8 = 0$

3. $2d^2 - 10d - 9 = 0$

4. $4x^2 = x + 4$

5. $3x^2 - 13x = 10$

6. $6x^2 - 10x + 10 = 0$

7. $2k^2 - 20 = -k$

8. $6p^2 = -11p - 40$

9. $9 - 18x + 9x^2 = 0$

10. $12x^2 + 9 = -6x$

11. $9a^2 = 81$

12. $16y^2 + 16y + 4 = 0$

13. $8x^2 + 9x = 2$

14. $4a^2 - 4a + 4 = 3$

15. $3b^2 - 18b = -14$