5.1 Solve Inequalities Using Addition and Subtraction

Before You solved equations using addition and subtraction.

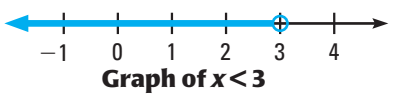
Now You will solve inequalities using addition and subtraction.

Why So you can describe desert temperatures, as in Example 1.

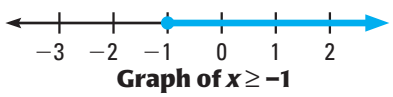
Key Vocabulary

- **graph of an inequality**
- **equivalent inequalities**
- **inequality**
- **solution of an inequality**

On a number line, the **graph of an inequality** in one variable is the set of points that represent all solutions of the inequality. To graph an inequality in one variable, use an open circle for $<$ or $>$ and a closed circle for $\leq$ or $\geq$. The graphs of $x < 3$ and $x \geq -1$ are shown below.



Graph of $x < 3$



Graph of $x \geq -1$

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

EXAMPLE 1 Write and graph an inequality

DEATH VALLEY The highest temperature recorded in the United States was 134°F at Death Valley, California, in 1913. Use only this fact to write and graph an inequality that describes the temperatures in the United States.

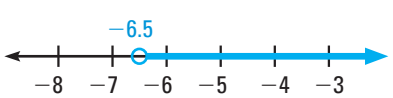
Solution

Let T represent a temperature (in degrees Fahrenheit) in the United States. The value of T must be less than or equal to 134. So, an inequality is $T \leq 134$.

EXAMPLE 2 Write inequalities from graphs

Write an inequality represented by the graph.

a.



b.

Solution

a. The open circle means that -6.5 is not a solution of the inequality. Because the arrow points to the right, all numbers greater than -6.5 are solutions.

▶ An inequality represented by the graph is $x > -6.5$.

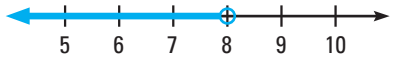
b. The closed circle means that 4 is a solution of the inequality. Because the arrow points to the left, all numbers less than 4 are solutions.

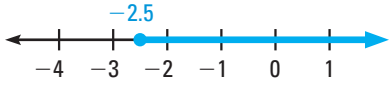
▶ An inequality represented by the graph is $x \leq 4$.

GUIDED PRACTICE for Examples 1 and 2

1. **ANTARCTICA** The lowest temperature recorded in Antarctica was −129°F at the Russian Vostok station in 1983. Use only this fact to write and graph an inequality that describes the temperatures in Antarctica.

Write an inequality represented by the graph.

2. 

3. 

EQUIVALENT INEQUALITIES Just as you used properties of equality to produce equivalent equations, you can use properties of inequality to produce *equivalent inequalities*. **Equivalent inequalities** are inequalities that have the same solutions.

KEY CONCEPT *For Your Notebook*

Addition Property of Inequality

Words Adding the same number to each side of an inequality produces an equivalent inequality.

Algebra If $a > b$, then $a + c > b + c$. If $a \geq b$, then $a + c \geq b + c$.

If $a < b$, then $a + c < b + c$. If $a \leq b$, then $a + c \leq b + c$.

EXAMPLE 3 Solve an inequality using addition

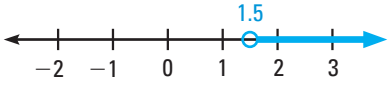
Solve $x - 5 > -3.5$. Graph your solution.

$x - 5 > -3.5$ Write original inequality.

$x - 5 + 5 > -3.5 + 5$ Add 5 to each side.

$x > 1.5$ Simplify.

▶ The solutions are all real numbers greater than 1.5. Check by substituting a number greater than 1.5 for x in the original inequality.



CHECK $x - 5 > -3.5$ Write original inequality.

$6 - 5 \stackrel{?}{>} -3.5$ Substitute 6 for x.

$1 > -3.5$ ✓ Solution checks.

GUIDED PRACTICE for Example 3

Solve the inequality. Graph your solution.

4. $x - 9 \leq 3$

5. $p - 9.2 < -5$

6. $-1 \geq m - \frac{1}{2}$

KEY CONCEPT — For Your Notebook

Subtraction Property of Inequality

Words Subtracting the same number from each side of an inequality produces an equivalent inequality.

Algebra If $a > b$, then $a - c > b - c$. If $a \geq b$, then $a - c \geq b - c$.

If $a < b$, then $a - c < b - c$. If $a \leq b$, then $a - c \leq b - c$.

EXAMPLE 4 Solve an inequality using subtraction

Solve $9 \geq x + 7$. Graph your solution.

$9 \geq x + 7$ — Write original inequality.

$9 - 7 \geq x + 7 - 7$ — Subtract 7 from each side.

$2 \geq x$ — Simplify.

▶ You can rewrite $2 \geq x$ as $x \leq 2$. The solutions are all real numbers less than or equal to 2.

Animated Algebra at my.hrw.com

EXAMPLE 5 Solve a real-world problem

READING
The phrase "no more than" indicates that you use the ≤ symbol.

LUGGAGE WEIGHTS You are checking a bag at an airport. Bags can weigh no more than 50 pounds. Your bag weighs 16.8 pounds. Find the possible weights w (in pounds) that you can add to the bag.

Solution

Write a verbal model. Then write and solve an inequality.

Weight of bag	+	Weight you can add	≤	Weight limit
↓		↓		↓
16.8	+	w	≤	**50**

$16.8 + w \leq 50$ — Write inequality.

$16.8 + w - 16.8 \leq 50 - 16.8$ — Subtract 16.8 from each side.

$w \leq 33.2$ — Simplify.

▶ You can add no more than 33.2 pounds.

GUIDED PRACTICE for Examples 4 and 5

7. Solve $y + 5.5 > 6$. Graph your solution.

8. WHAT IF? In Example 5, suppose your bag weighs 29.1 pounds. Find the possible weights (in pounds) that you can add to the bag.

5.1 EXERCISES

HOMEWORK KEY

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 7, 15, and 33
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 29, 34, 35, and 38
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 37

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: To graph $x < -8$, you draw a(n) ___?___ circle at -8, and you draw an arrow to the ___?___.

2. ★ **WRITING** Are $x + 7 \geq 18$ and $x \geq 25$ equivalent inequalities? *Explain*.

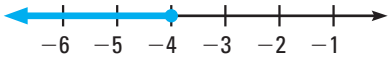
EXAMPLE 1 for Exs. 3–5

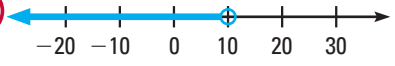
WRITING AND GRAPHING INEQUALITIES **Write and graph an inequality that describes the situation.**

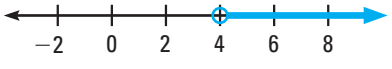
3. The speed limit on a highway is 60 miles per hour.
4. You must be at least 16 years old to go on a field trip.
5. A child must be taller than 48 inches to get on an amusement park ride.

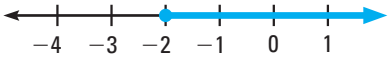
EXAMPLE 2 for Exs. 6–9

WRITING INEQUALITIES **Write an inequality represented by the graph.**

6. 

7. 

8. 

9. 

EXAMPLES 3 and 4 for Exs. 10–23

SOLVING INEQUALITIES **Solve the inequality. Graph your solution.**

10. $x + 4 < 5$	11. $-8 \leq 8 + y$	12. $-1\frac{1}{4} \leq m + 3$	13. $n + 17 \leq 16\frac{4}{5}$
14. $8.2 + v > -7.6$	15. $w + 14.9 > -2.7$	16. $r - 4 < -5$	17. $1 \leq s - 8$
18. $-1\frac{1}{3} \leq p - 8\frac{1}{3}$	19. $q - 1\frac{1}{3} > -2\frac{1}{2}$	20. $2.1 \geq c - 6.7$	21. $d - 1.92 > -8.76$

ERROR ANALYSIS ***Describe*** **and correct the error in solving the inequality or in graphing the solution.**

22. $x + 8 < -3$

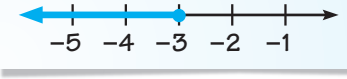
$x + 8 - 8 < -3 + 8$

$x < 5$

23. $-17 \leq x - 14$

$-17 + 14 \leq x - 14 + 14$

$-3 \leq x$

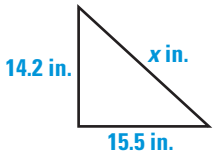


TRANSLATING SENTENCES **Write the verbal sentence as an inequality. Then solve the inequality and graph your solution.**

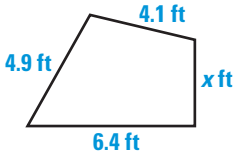
24. The sum of 11 and m is greater than -23.
25. The difference of n and 15 is less than or equal to 37.
26. The difference of c and 13 is less than -19.

GEOMETRY **Write and solve an inequality to find the possible values of x.**

27. Perimeter < 51.3 inches



28. Perimeter ≤ 18.7 feet



29. ★ **WRITING** Is it possible to check all the numbers that are solutions of an inequality? Does checking one solution guarantee that you have solved an inequality correctly? *Explain* your answers.

30. **CHALLENGE** Write and graph an inequality that represents the numbers that are *not* solutions of $x - 12 \geq 5.7$.

PROBLEM SOLVING

EXAMPLE 5 for Exs. 31–35

31. **INTERNET** You earn points from buying items at an Internet shopping site. You would like to redeem 2350 points to get an item for free, but you want to be sure to have more than 6000 points left over. What are the possible numbers of points you can have before making a redemption?

32. **SPORTS RECORDS** In 1982 Wayne Gretsky set a new record for the greatest number of hockey goals in one season with 92 goals. Suppose that a hockey player has 59 goals so far in a season. What are the possible numbers of additional goals that the player can make in order to match or break Wayne Gretsky's record?

33. **MULTI-STEP PROBLEM** In aerial ski competitions, athletes perform two acrobatic ski jumps, and the scores on both jumps are added together. The table shows your competitor's first and second scores and your first score.

Ski jump	Competitor's score	Your score
1	127.04	129.49
2	129.98	?

a. Write and solve an inequality to find the scores s that you can earn in your second jump in order to beat your competitor.

b. Will you beat your competitor if you earn 128.13 points? 126.78 points? 127.53 points? *Justify* your answers.

34. ★ **MULTIPLE CHOICE** You want to buy a jacket at a clothing store, and you can spend at most \$30. You have a coupon for \$3 off any item at the store. Which inequality can you use to find the original prices p of jackets that you can buy?

Ⓐ $3 + p \geq 30$ Ⓑ $30 + p \leq 3$ Ⓒ $p - 3 \leq 30$ Ⓓ $p - 30 \geq 3$

35. ★ **OPEN-ENDED** *Describe* a real-world situation that can be modeled by the inequality $x + 14 \geq 17$. *Explain* what the solution of the inequality means in this situation.

36. VEHICLE WEIGHTS According to a state law for vehicles traveling on state roads, the maximum total weight of the vehicle and its contents depends on the number of axles the vehicle has.

Maximum Total Weights

For each type of vehicle, write and solve an inequality to find the possible weights w (in pounds) of a vehicle when its contents weigh 14,200 pounds. Can a vehicle that has 2 axles and weighs 20,000 pounds hold 14,200 pounds of contents? *Explain*.

37. MULTIPLE REPRESENTATIONS Your friend is willing to spend no more than $17,000 for a new car. The car dealership offers $3000 cash back for the purchase of a new car.

a. Making a Table Make a table of values that gives the final price y of a car after the cash back offer is applied to the original price x. Use the following values for x: 19,459, 19,989, 20,549, 22,679, 23,999.

b. Writing an Inequality Write and solve an inequality to find the original prices of the cars that your friend will consider buying.

38. ★ SHORT RESPONSE A 4-member track team is trying to match or beat last year's winning time of 3 minutes 41.1 seconds for a 1600 meter relay race. The table shows the 400 meter times for the first three athletes.

Athlete	Time (sec)
1	53.34
2	56.38
3	57.46

a. Calculate What are the possible times that the last athlete can run 400 meters in order for the team to match or beat last year's time?

b. Decide So far this season the last athlete's fastest 400 meter time is 53.18 seconds, and his average 400 meter time is 53.92 seconds. In this race the last athlete expects to run faster than his slowest time this season. Is it possible for the team to fail to meet its goal? *Explain*.

39. CHALLENGE A public television station wants to raise at least $72,000 in a pledge drive. The station raised an average of $5953 per day for the first 3 days and an average of $6153 per day for the next 3 days. What are the possible additional amounts that the station can raise to meet its goal?

Inequalities with Negative Coefficients

MATERIALS • index cards

MATHEMATICAL PRACTICES
Reason abstractly and quantitatively.

QUESTION How do you solve an inequality with a negative coefficient?

EXPLORE Check solutions of inequalities

STEP 1 ***Write integers*** Write the integers from -5 to 5 on index cards. Place the cards face up as shown.

−5	−4	−3	−2	−1	0	1	2	3	4	5

STEP 2 ***Check solutions*** Determine whether each integer is a solution of $4x \geq 8$. If the integer is *not* a solution, turn over the card.

							2	3	4	5

STEP 3 ***Check solutions*** Turn all the cards face up. Repeat Step 2 for $-4x \geq 8$.

−5	−4	−3	−2							

DRAW CONCLUSIONS Use your observations to complete these exercises

1. State an operation that you can perform on both sides of $4x \geq 8$ to obtain the solutions found in Step 2. Then solve the inequality.
2. Copy and complete the steps below for solving $-4x \geq 8$.

   $-4x \geq 8$ Write original inequality.

   ___?___ Add $4x$ to each side.

   ___?___ Subtract 8 from each side.

   ___?___ Divide each side by 4.

   ___?___ Rewrite inequality with x on the left side.
3. Does dividing both sides of $-4x \geq 8$ by -4 give the solution found in Exercise 2? If not, what else must you do to the inequality when you divide by -4?
4. Do you need to change the direction of the inequality symbol when you divide each side of an inequality by a positive number? by a negative number?

Solve the inequality.

5. $20x \geq 5$ **6.** $-9x \leq 45$ **7.** $-8x > 40$ **8.** $7x < 21$