

5.2 Solve Inequalities Using Multiplication and Division



Before

You solved inequalities using addition and subtraction.

Now

You will solve inequalities using multiplication and division.

Why?

So you can find possible distances traveled, as in Ex. 40.

Key Vocabulary

- inequality
- equivalent inequalities

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solving an inequality using multiplication is similar to solving an equation using multiplication, but it is different in an important way.

KEY CONCEPT

For Your Notebook

Multiplication Property of Inequality

Words Multiplying each side of an inequality by a *positive* number produces an equivalent inequality.

Multiplying each side of an inequality by a *negative* number and *reversing the direction of the inequality symbol* produces an equivalent inequality.

Algebra If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c < 0$, then $ac > bc$.
If $a > b$ and $c > 0$, then $ac > bc$. If $a > b$ and $c < 0$, then $ac < bc$.

This property is also true for inequalities involving \leq and \geq .

EXAMPLE 1 Solve an inequality using multiplication

Solve $\frac{x}{4} < 5$. Graph your solution.

$$\frac{x}{4} < 5 \quad \text{Write original inequality.}$$

$$4 \cdot \frac{x}{4} < 4 \cdot 5 \quad \text{Multiply each side by 4.}$$

$$x < 20 \quad \text{Simplify.}$$

► The solutions are all real numbers less than 20. Check by substituting a number less than 20 in the original inequality.



GUIDED PRACTICE for Example 1

Solve the inequality. Graph your solution.

1. $\frac{x}{3} > 8$

2. $\frac{m}{8} \leq -2$

3. $\frac{y}{2.5} \geq -4$

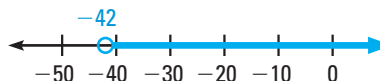
EXAMPLE 2 Solve an inequality using multiplicationSolve $\frac{x}{-6} < 7$. Graph your solution.

$$\frac{x}{-6} < 7 \quad \text{Write original inequality.}$$

$$\rightarrow -6 \cdot \frac{x}{-6} > -6 \cdot 7 \quad \text{Multiply each side by } -6. \text{ Reverse inequality symbol.}$$

$$x > -42 \quad \text{Simplify.}$$

▶ The solutions are all real numbers greater than -42 . Check by substituting a number greater than -42 in the original inequality.

**AVOID ERRORS**

Because you are multiplying by a negative number, be sure to reverse the inequality symbol.

$$\text{CHECK } \frac{x}{-6} < 7 \quad \text{Write original inequality.}$$

$$\frac{0}{-6} < 7 \quad \text{Substitute 0 for } x.$$

$$0 < 7 \quad \checkmark \quad \text{Solution checks.}$$

USING DIVISION The rules for solving an inequality using division are similar to the rules for solving an inequality using multiplication.

KEY CONCEPT*For Your Notebook***Division Property of Inequality**

Words Dividing each side of an inequality by a *positive* number produces an equivalent inequality.

Dividing each side of an inequality by a *negative* number and *reversing the direction of the inequality symbol* produces an equivalent inequality.

Algebra If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$. If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

This property is also true for inequalities involving \leq and \geq .

EXAMPLE 3 Solve an inequality using divisionSolve $-3x > 24$.

$$-3x > 24 \quad \text{Write original inequality.}$$

$$\frac{-3x}{-3} < \frac{24}{-3} \quad \text{Divide each side by } -3. \text{ Reverse inequality symbol.}$$

$$x < -8 \quad \text{Simplify.}$$

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**GUIDED PRACTICE** for Examples 2 and 3

Solve the inequality. Graph your solution.

4. $\frac{x}{-4} > 12$

5. $\frac{m}{-7} < 1.6$

6. $5v \geq 45$

7. $-6n < 24$

**EXAMPLE 4** Standardized Test Practice

A student pilot plans to spend 80 hours on flight training to earn a private license. The student has saved \$6000 for training. Which inequality can you use to find the possible hourly rates r that the student can afford to pay for training?

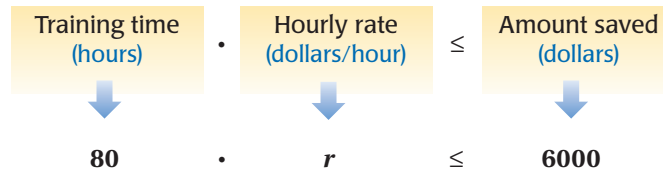
- (A) $80r \geq 6000$ (B) $80r \leq 6000$ (C) $6000r \geq 80$ (D) $6000r \leq 80$

Solution

The total cost of training can be at most the amount of money that the student has saved. Write a verbal model for the situation. Then write an inequality.

ELIMINATE CHOICES

You need to multiply the hourly rate and the number of hours, which is 80, not 6000. So, you can eliminate choices C and D.



▶ The correct answer is B. (A) (B) (C) (D)

EXAMPLE 5 Solve a real-world problem

PILOTING In Example 4, what are the possible hourly rates that the student can afford to pay for training?

Solution

$$80 \cdot r \leq 6000 \quad \text{Write inequality.}$$

$$\frac{80r}{80} \leq \frac{6000}{80} \quad \text{Divide each side by 80.}$$

$$r \leq 75 \quad \text{Simplify.}$$

▶ The student can afford to pay at most \$75 per hour for training.

**GUIDED PRACTICE** for Examples 4 and 5

8. **WHAT IF?** In Example 5, suppose the student plans to spend 90 hours on flight training and has saved \$6300. Write and solve an inequality to find the possible hourly rates that the student can afford to pay for training.

5.2 EXERCISES

HOMework KEY

- = See **WORKED-OUT SOLUTIONS**
Exs. 5, 9, and 39
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 27, 34, and 41
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 38

SKILL PRACTICE

1. **VOCABULARY** Which property are you using when you solve $5x \geq 30$ by dividing each side by 5?
2. ★ **WRITING** Are $\frac{x}{-4} < -9$ and $x < 36$ equivalent inequalities? *Explain* your answer.

EXAMPLES
1, 2, and 3
for Exs. 3–29


SOLVING INEQUALITIES Solve the inequality. Graph your solution.

- | | | | |
|---------------------------|------------------------------|--|-------------------------------|
| 3. $2p \geq 14$ | 4. $\frac{x}{-3} < -10$ | 5. $-6y < -36$ | 6. $40 > \frac{w}{5}$ |
| 7. $\frac{q}{4} < 7$ | 8. $72 \leq 9r$ | 9. $\frac{g}{6} > -20$ | 10. $-11m \leq -22$ |
| 11. $-90 \geq 4t$ | 12. $\frac{n}{3} < -9$ | 13. $60 \leq -12s$ | 14. $\frac{v}{-4} \geq -8$ |
| 15. $-8.4f > 2.1$ | 16. $\frac{d}{-2} \leq 18.6$ | 17. $9.6 < -16c$ | 18. $0.07 \geq \frac{k}{7}$ |
| 19. $-1.5 \geq 6z$ | 20. $\frac{x}{-5} \leq -7.5$ | 21. $1.02 < -3j$ | 22. $\frac{y}{-4.5} \geq -10$ |
| 23. $\frac{r}{-30} < 1.8$ | 24. $1.9 \leq -5p$ | 25. $\frac{m}{0.6} > -40$ | 26. $-2t > -1.22$ |


27. ★ **WRITING** How is solving $ax > b$ where $a > 0$ similar to solving $ax > b$ where $a < 0$? How is it different?

ERROR ANALYSIS Describe and correct the error in solving the inequality.

28.

$$\begin{aligned} -15x &> 45 \\ \frac{-15x}{-15} &> \frac{45}{-15} \\ x &> -3 \end{aligned}$$


29.

$$\begin{aligned} \frac{x}{9} &\leq -7 \\ 9 \cdot \frac{x}{9} &\leq 9 \cdot (-7) \\ x &\geq -63 \end{aligned}$$


TRANSLATING SENTENCES In Exercises 30–33, write the verbal sentence as an inequality. Then solve the inequality and graph your solution.

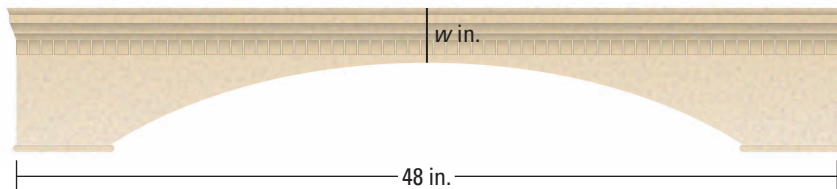
30. The product of 8 and x is greater than 50.
31. The product of -15 and y is less than or equal to 90.
32. The quotient of v and -9 is less than -18 .
33. The quotient of w and 24 is greater than or equal to $-\frac{1}{6}$.
34. ★ **OPEN-ENDED** Write an inequality in the form $ax < b$ such that the solutions are all real numbers greater than 4.

35. **CHALLENGE** For the given values of a and b , tell whether the solution of $ax > b$ consists of *positive numbers*, *negative numbers*, or *both*. *Explain.*
- a. $a < 0, b > 0$ b. $a > 0, b > 0$ c. $a > 0, b < 0$ d. $a < 0, b < 0$

PROBLEM SOLVING

EXAMPLES
4 and 5
Exs. 36–39

36. **MUSIC** You have \$90 to buy CDs for your friend's party. The CDs cost \$18 each. What are the possible numbers of CDs that you can buy?
37. **JOB SKILLS** You apply for a job that requires the ability to type 40 words per minute. You practice typing on a keyboard for 5 minutes. The average number of words you type per minute must at least meet the job requirement. What are the possible numbers of words that you can type in 5 minutes in order to meet or exceed the job requirement?
38. **MULTIPLE REPRESENTATIONS** You are stacking books on a shelf that has a height of 66 centimeters. Each book has a thickness of 4 centimeters.
- Using a Model** Use a concrete model to find the possible numbers of books that you can stack as follows: Cut strips of paper 4 centimeters wide to represent the books. Then place the strips one above the other until they form a column no taller than 66 centimeters.
 - Writing an Inequality** Write and solve an inequality to find the possible numbers of books that you can stack.
 - Drawing a Graph** Write and graph an equation that gives the height y of stacked books as a function of the number x of books. Then graph $y = 66$ in the same coordinate plane. To find the possible numbers of books that you can stack, identify the integer x -coordinates of the points on the first graph that lie *on or below* the graph of $y = 66$.
 - Choosing a Method** Suppose the shelf has a height of 100 centimeters. Which method would you use to find the possible numbers of books, *a concrete model*, *solving an inequality*, or *drawing a graph*? *Explain.*
39. **MANUFACTURING** A manufacturer of architectural moldings recommends that the length of a piece be no more than 15 times its minimum width w (in inches) in order to prevent cracking. For the piece shown, what could the values of w be?



40. **RECREATION** A water-skiing instructor recommends that a boat pulling a beginning skier have a speed less than 18 miles per hour. Write and solve an inequality that you can use to find the possible distances d (in miles) that a beginner can travel in 45 minutes of practice time.

41. ★ **EXTENDED RESPONSE** A state agency that offers wild horses for adoption requires that a potential owner reserve 400 square feet of land per horse in a corral.
- Solve** A farmer has a rectangular corral whose length is 80 feet and whose width is 82 feet. Write and solve an inequality to find the possible numbers h of horses that the corral can hold.
 - Explain** If the farmer increases the length and width of the corral by 20 feet each, will the corral be able to hold only 1 more horse? *Explain* your answer without calculating the new area of the corral.
 - Calculate** The farmer decides to increase the length and width of the corral by 15 feet each. Find the possible numbers of horses that the corral can hold. Your answer should include the following:
 - a calculation of the new area of the corral
 - a description of your steps for solving the problem
42. **CHALLENGE** An electronics store is selling a laptop computer for \$1050. You can spend no more than \$900 for the laptop, so you wait for it to go on sale. Also, you plan to use a store coupon for 5% off the sale price. For which decreases in price will you consider buying the laptop?



QUIZ

Solve the inequality. Graph your solution.

1. $x + 8 \geq -5$

2. $y + 6 < 14$

3. $-8 \leq v - 5$

4. $w - 11 > 2$

5. $-40 < -5r$

6. $-93 < 3s$

7. $-2m \geq 26$

8. $\frac{n}{-4} > -7$

9. $\frac{c}{6} \leq -8$

10. **FOOD PREPARATION** You need to make at least 150 sandwiches for a charity event. You can make 3 sandwiches per minute. How long will it take you to make the number of sandwiches you need?

