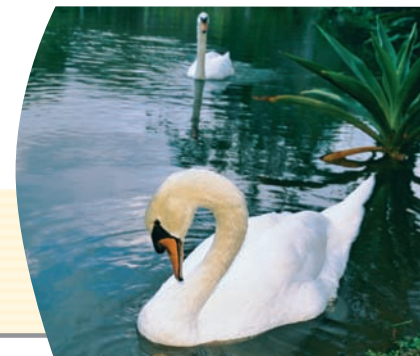


# 5.3 Solve Multi-Step Inequalities



**Before**

You solved one-step inequalities.

**Now**

You will solve multi-step inequalities.

**Why?**

So you can compare animal habitats, as in Ex. 39.

## Key Vocabulary

• inequality

The steps for solving two-step and multi-step equations can be applied to linear inequalities. For inequalities, be sure to reverse the inequality symbol when multiplying or dividing by a negative number.

COMMON CORE

**CC.9-12.A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

### EXAMPLE 1 Solve a two-step inequality

Solve  $3x - 7 < 8$ . Graph your solution.

$$3x - 7 < 8 \quad \text{Write original inequality.}$$

$$3x < 15 \quad \text{Add 7 to each side.}$$

$$x < 5 \quad \text{Divide each side by 3.}$$

► The solutions are all real numbers less than 5. Check by substituting a number less than 5 in the original inequality.



**CHECK**  $3x - 7 < 8$  Write original inequality.

$$3(0) - 7 < 8 \quad \text{Substitute 0 for } x.$$

$$-7 < 8 \quad \text{Solution checks.}$$

### EXAMPLE 2 Solve a multi-step inequality

Solve  $-0.6(x - 5) \leq 15$ .

$$-0.6(x - 5) \leq 15 \quad \text{Write original inequality.}$$

$$-0.6x + 3 \leq 15 \quad \text{Distributive property}$$

$$-0.6x \leq 12 \quad \text{Subtract 3 from each side.}$$

$$x \geq -20 \quad \text{Divide each side by } -0.6. \text{ Reverse inequality symbol.}$$



### GUIDED PRACTICE for Examples 1 and 2

Solve the inequality. Graph your solution.

1.  $2x - 5 \leq 23$

2.  $-6y + 5 \leq -16$

3.  $-\frac{1}{4}(p - 12) > -2$

**EXAMPLE 3** Solve a multi-step inequality**ANOTHER WAY**

You can also solve the inequality by subtracting 17 and  $6x$  from each side, as follows:

$$6x - 7 > 2x + 17$$

$$6x - 24 > 2x$$

$$-24 > -4x$$

$$6 < x$$

The inequality  $6 < x$  is equivalent to  $x > 6$ .

Solve  $6x - 7 > 2x + 17$ . Graph your solution.

$$6x - 7 > 2x + 17 \quad \text{Write original inequality.}$$

$$6x > 2x + 24 \quad \text{Add 7 to each side.}$$

$$4x > 24 \quad \text{Subtract } 2x \text{ from each side.}$$

$$x > 6 \quad \text{Divide each side by 4.}$$

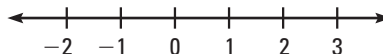
▶ The solutions are all real numbers greater than 6.



**NUMBER OF SOLUTIONS** If an inequality is equivalent to an inequality that is true, such as  $-3 < 0$ , then the solutions of the inequality are *all real numbers*. If an inequality is equivalent to an inequality that is false, such as  $4 < -1$ , then the inequality has *no solution*.



**Graph of an inequality whose solutions are all real numbers**



**Graph of an inequality that has no solution**

**EXAMPLE 4** Identify the number of solutions of an inequality

Solve the inequality, if possible.

a.  $14x + 5 < 7(2x - 3)$

b.  $12x - 1 > 6(2x - 1)$

**Solution**

a.  $14x + 5 < 7(2x - 3)$  **Write original inequality.**

$$14x + 5 < 14x - 21 \quad \text{Distributive property}$$

$$5 < -21 \quad \text{Subtract } 14x \text{ from each side.}$$

▶ There are no solutions because  $5 < -21$  is false.

b.  $12x - 1 > 6(2x - 1)$  **Write original inequality.**

$$12x - 1 > 12x - 6 \quad \text{Distributive property}$$

$$-1 > -6 \quad \text{Subtract } 12x \text{ from each side.}$$

▶ All real numbers are solutions because  $-1 > -6$  is true.

**GUIDED PRACTICE** for Examples 3 and 4

Solve the inequality, if possible. Graph your solution.

4.  $5x - 12 \leq 3x - 4$

5.  $5(m + 5) < 5m + 17$

6.  $1 - 8s \leq -4(2s - 1)$



## EXAMPLE 5 Solve a multi-step problem

**CAR WASH** Use the sign shown. A gas station charges \$.10 less per gallon of gasoline if a customer also gets a car wash. What are the possible amounts (in gallons) of gasoline that you can buy if you also get a car wash and can spend at most \$20?



### ANOTHER WAY

For an alternative method for solving the problem in Example 5, see the **Problem Solving Workshop**.

### Solution

Because you are getting a car wash, you will pay  $\$2.09 - \$.10 = \$1.99$  per gallon of gasoline. Let  $g$  be the amount (in gallons) of gasoline that you buy.

**STEP 1** Write a verbal model. Then write an inequality.

Price of gasoline (dollars/gallon)	•	Amount of gasoline (gallons)	+	Price of car wash (dollars)	≤	Maximum amount (dollars)
↓		↓		↓		↓
1.99	•	$g$	+	8	≤	20

**STEP 2** Solve the inequality.

$$1.99g + 8 \leq 20$$

Write inequality.

$$1.99g \leq 12$$

Subtract 8 from each side.

$$g \leq 6.03015 \dots$$

Divide each side by 1.99.

▶ You can buy up to slightly more than 6 gallons of gasoline.

**CHECK** You can use a table to check the reasonableness of your answer.

The table shows that you will pay \$19.94 for exactly 6 gallons of gasoline. Because \$19.94 is less than \$20, it is reasonable to conclude that you can buy slightly more than 6 gallons of gasoline.

Gasoline (gal)	Total amount spent (dollars)
0	8.00
1	9.99
2	11.98
3	13.97
4	15.96
5	17.95
6	19.94



### GUIDED PRACTICE for Example 5

7. **WHAT IF?** In Example 5, suppose that a car wash costs \$9 and gasoline regularly costs \$2.19 per gallon. What are the possible amounts (in gallons) of gasoline that you can buy?

8. **CAMP COSTS** You are saving money for a summer camp that costs \$1800. You have saved \$500 so far, and you have 14 more weeks to save the total amount. What are the possible average amounts of money that you can save per week in order to have a total of at least \$1800 saved?

# 5.3 EXERCISES

## HOMEWORK KEY

- = See **WORKED-OUT SOLUTIONS**  
Exs. 5, 19, and 39
- ★ = **STANDARDIZED TEST PRACTICE**  
Exs. 2, 33, 39, 40, and 42
- ◆ = **MULTIPLE REPRESENTATIONS**  
Ex. 41

### SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The inequalities  $3x - 1 < 11$ ,  $3x < 12$ , and  $x < 4$  are called    ?.
2. ★ **WRITING** How do you know whether an inequality has no solutions? How do you know whether the solutions are all real numbers?

#### EXAMPLES

1, 2, and 3

for Exs. 3–16

#### SOLVING INEQUALITIES Solve the inequality. Graph your solution.

- |                                    |  |  |
|------------------------------------|--|--|
| 3. $2x - 3 > 7$                    | 4. $5y + 9 \leq 4$                         | <span style="border: 1px solid red; border-radius: 50%; padding: 2px;">5.</span> $8v - 3 \geq -11$ |
| 6. $3(w + 12) < 0$                 | 7. $7(r - 3) \geq -13$                     | 8. $2(s + 4) \leq 16$  |
| 9. $4 - 2m > 7 - 3m$               | 10. $8n - 2 > 17n + 9$                     | 11. $-10p > 6p - 8$  |
| 12. $4 - \frac{1}{2}q \leq 33 - q$ | 13. $-\frac{2}{3}d - 2 < \frac{1}{3}d + 8$ | 14. $8 - \frac{4}{5}f > -14 - 2f$  |

#### ERROR ANALYSIS Describe and correct the error in solving the inequality.

15.

$$\begin{aligned} 17 - 3x &\geq 56 \\ -3x &\geq 39 \\ x &\geq -13 \end{aligned}$$



16.

$$\begin{aligned} -4(2x - 3) &< 28 \\ -8x - 12 &< 28 \\ -8x &< 40 \\ x &> -5 \end{aligned}$$



#### EXAMPLE 4

for Exs. 17–28

#### SOLVING INEQUALITIES Solve the inequality, if possible.

- |                                      |   |  |
|--------------------------------------|---|--|
| 17. $3p - 5 > 2p + p - 7$            | 18. $5d - 8d - 4 \leq -4 + 3d$                | <span style="border: 1px solid red; border-radius: 50%; padding: 2px;">19.</span> $3(s - 4) \geq 2(s - 6)$ |
| 20. $2(t - 3) > 2t - 8$              | 21. $5(b + 9) \leq 5b + 45$                   | 22. $2(4c - 7) \geq 8(c - 3)$  |
| 23. $6(x + 3) < 5x + 18 + x$         | 24. $4 + 9y - 3 \geq 3(3y + 2)$               | 25. $2.2h + 0.4 \leq 2(1.1h - 0.1)$  |
| 26. $9.5j - 6 + 5.5j \geq 3(5j - 2)$ | 27. $\frac{1}{5}(4m + 10) < \frac{4}{5}m + 2$ | 28. $\frac{3}{4}(8n - 4) < -3(1 - 2n)$   |

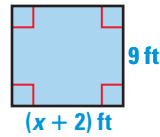
#### TRANSLATING PHRASES Translate the verbal phrase into an inequality. Then solve the inequality and graph your solution.

29. Four more than the product of 3 and  $x$  is less than 40.
30. Twice the sum of  $x$  and 8 is greater than or equal to  $-36$ .
31. The sum of  $5x$  and  $2x$  is greater than the difference of  $9x$  and 4.
32. The product of 6 and the difference of  $6x$  and 3 is less than or equal to the product of  $-2$  and the sum of 4 and  $8x$ .
33. ★ **MULTIPLE CHOICE** For which values of  $a$  and  $b$  are all the solutions of  $ax + b > 0$  positive?
 

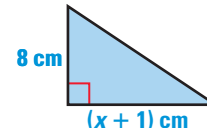
<span style="border: 1px solid black; border-radius: 50%; padding: 2px;">A</span> $a > 0, b > 0$	<span style="border: 1px solid black; border-radius: 50%; padding: 2px;">B</span> $a < 0, b < 0$	<span style="border: 1px solid black; border-radius: 50%; padding: 2px;">C</span> $a > 0, b < 0$	<span style="border: 1px solid black; border-radius: 50%; padding: 2px;">D</span> $a < 0, b = 0$
--	--	--	--

**GEOMETRY** Write and solve an inequality to find the possible values of  $x$ .

34. Area  $> 81$  square feet



35. Area  $\leq 44$  square centimeters



36. **CHALLENGE** For which value of  $a$  are all the solutions of  $2(x - 5) \geq 3x + a$  less than or equal to 5?

**PROBLEM SOLVING**

**EXAMPLE 5**  
for Exs. 37–40

37. **CD BURNING** A blank CD can hold 70 minutes of music. So far you have burned 25 minutes of music onto the CD. You estimate that each song lasts 4 minutes. What are the possible numbers of additional songs that you can burn onto the CD?

38. **BUSINESS** You spend \$46 on supplies to make wooden ornaments and plan to sell the ornaments for \$8.50 each. What are the possible numbers of ornaments that you can sell in order for your profit to be positive?

39. **★ SHORT RESPONSE** A zookeeper is designing a rectangular habitat for swans, as shown. The zookeeper needs to reserve 500 square feet for the first 2 swans and 125 square feet for each additional swan.



a. **Calculate** What are the possible numbers of swans that the habitat can hold? *Explain* how you got your answer.

b. **Compare** Suppose that the zookeeper increases both the length and width of the habitat by 20 feet. What are the possible numbers of additional swans that the habitat can hold?

40. **★ MULTIPLE CHOICE** A gym is offering a trial membership for 3 months by discounting the regular monthly rate by \$50. You will consider joining the gym if the total cost of the trial membership is less than \$100. Which inequality can you use to find the possible regular monthly rates that you are willing to pay?

(A)  $3x - 50 < 100$

(B)  $3x - 50 > 100$

(C)  $3(x - 50) < 100$

(D)  $3(x - 50) > 100$

41. **MULTIPLE REPRESENTATIONS** A baseball pitcher makes 53 pitches in the first four innings of a game and plans to pitch in the next 3 innings.
- Making a Table** Make a table that gives the total number  $t$  of pitches made if the pitcher makes an average of  $p$  pitches per inning in the next 3 innings. Use the following values for  $p$ : 15, 16, 17, 18, 19.
  - Writing an Inequality** The baseball coach assigns a maximum of 105 pitches to the pitcher for the game. Write and solve an inequality to find the possible average numbers of pitches that the pitcher can make in each of the next three innings.
42. **★ EXTENDED RESPONSE** A state imposes a sales tax on items of clothing that cost more than \$175. The tax applies only to the difference of the price of the item and \$175.
- Calculate** Use the receipt shown to find the tax rate (as a percent). *Explain* how you got your answer.
  - Apply** A shopper has \$400 to spend on a winter coat. Write and solve an inequality to find the prices  $p$  of coats that the shopper can afford. Assume that  $p \geq 175$ .
  - Compare** Another state imposes a 4% sales tax on the entire price of an item of clothing. For which prices would paying the 4% tax be cheaper than paying the tax described above? Your answer should include the following:
    - writing and solving an inequality that describes the situation
    - checking the reasonableness of your answer using one of the solutions of the inequality
43. **CHALLENGE** Your scores in four bowling league tournaments are 157, 161, 149, and 172. After the next game, you want your average score to be at least 167. What are the possible scores that you can earn in your next tournament in order to meet your goal?

THE STYLE STORE	
Item:	Price: \$ 300.00
Suit	Tax: \$ 6.25
	Total: \$ 306.25

**Another Way to Solve Example 5**



Make sense of problems and persevere in solving them.

**PROBLEM**

**CAR WASH** Use the sign shown. A gas station charges \$.10 less per gallon of gasoline if a customer also gets a car wash. What are the possible amounts (in gallons) of gasoline that you can buy if you also get a car wash and can spend at most \$20?



**METHOD 1**

**Work backward** One alternative approach is to work backward.

**STEP 1** Read the problem. It gives you the following information:

- amount you can spend: up to \$20
- price of a car wash: \$8
- regular price per gallon of gasoline: \$2.09
- discount per gallon of gasoline when you get a car wash: \$.10

Because you are getting a car wash, gasoline costs  $2.09 - .10$ , or \$1.99, per gallon.

**STEP 2** Work backward.

- Start with the amount you have to spend: \$20.
- Subtract the cost of a car wash:  $20 - 8 = 12$ .
- Make a table of values showing the amount of money you have left after buying various amounts of gasoline.

Gasoline (gal)	Amount of money left
0	\$12.00
1	\$10.01
2	\$8.02
3	\$6.03
4	\$4.04
5	\$2.05
6	\$.06

$- \$1.99$   
 $- \$1.99$   
 $- \$1.99$   
 $- \$1.99$   
 $- \$1.99$   
 $- \$1.99$

► You can buy up to slightly more than 6 gallons of gasoline.

**METHOD 2**

**Using a graph** Another alternative approach is to use a graph.

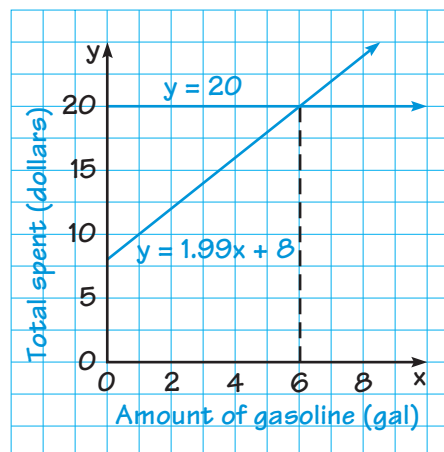
**STEP 1 Write** a verbal model. Then write an equation that gives the total amount of money  $y$  (in dollars) that you spend as a function of the amount  $x$  (in gallons) of gasoline that you buy.

Total spent (dollars)	=	Price of gasoline (dollars/gallon)	•	Amount of gasoline (gallons)	+	Price of car wash (dollars)
$y$	=	1.99	•	$x$	+	8

**STEP 2 Graph**  $y = 1.99x + 8$ .

**STEP 3 Graph**  $y = 20$  in the same coordinate plane. This equation gives the maximum amount of money that you can spend for gasoline and a car wash.

**STEP 4 Analyze** the graphs. The point of intersection shows that you can buy slightly more than 6 gallons of gasoline when you spend \$20. Because you can spend *at most* \$20, the solutions are the  $x$ -coordinates of the points on the graph of  $y = 1.99x + 8$  that lie *on or below* the graph of  $y = 20$ .



► You can buy up to slightly more than 6 gallons of gasoline.

**PRACTICE**

1. **BAKING** You need to bake at least 100 cookies for a bake sale. You can bake 12 cookies per batch of dough. What are the possible numbers of batches that will allow you to bake enough cookies? Solve this problem using two different methods.
2. **VIDEO GAMES** A video game console costs \$259, and games cost \$29 each. You saved \$400 to buy a console and games. What are the possible numbers of games that you can buy? Solve this problem using two different methods.
3. **WHAT IF?** In Exercise 2, suppose that you saved \$500 and decide to buy a video game console that costs \$299. What are the possible numbers of games that you can buy?
4. **MONEY** You need to have at least \$100 in your checking account to avoid a low balance fee. You have \$247 in your account, and you make withdrawals of \$20 per week. What are the possible numbers of weeks that you can withdraw money and avoid paying the fee? Solve this problem using two different methods.
5. **RUNNING TIMES** You are running a 10 mile race. You run the first 3 miles in 24.7 minutes. Your goal is to finish the race in less than 1 hour 20 minutes. What should your average running time (in minutes per mile) be for the remaining miles?