

5.4 Solve Compound Inequalities



Before

You solved one-step and multi-step inequalities.

Now

You will solve compound inequalities.

Why?

So you can describe possible heights, as in Example 2.

Key Vocabulary

• **compound inequality**

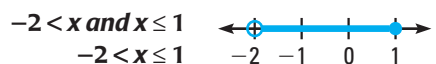
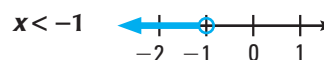
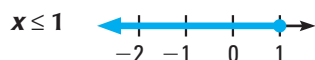
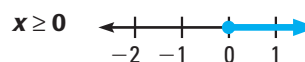
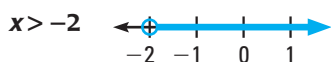
A **compound inequality** consists of two separate inequalities joined by *and* or *or*.

The graph of a compound inequality with *and* is the *intersection* of the graphs of the inequalities.

The graph of a compound inequality with *or* is the *union* of the graphs of the inequalities.



CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.



EXAMPLE 1 Write and graph compound inequalities

Translate the verbal phrase into an inequality. Then graph the inequality.

- a. All real numbers that are greater than -2 *and* less than 3

Inequality: $-2 < x < 3$



- b. All real numbers that are less than 0 *or* greater than or equal to 2

Inequality: $x < 0$ *or* $x \geq 2$



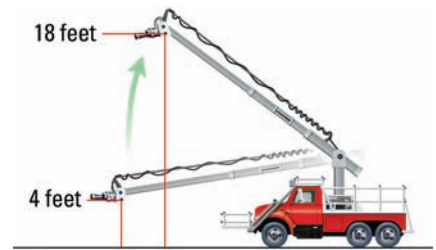
GUIDED PRACTICE for Example 1

Translate the verbal phrase into an inequality. Then graph the inequality.

- All real numbers that are less than -1 *or* greater than or equal to 4
- All real numbers that are greater than or equal to -3 *and* less than 5

EXAMPLE 2 Write and graph a real-world compound inequality

CAMERA CARS A crane sits on top of a camera car and faces toward the front. The crane's maximum height and minimum height above the ground are shown. Write and graph a compound inequality that describes the possible heights of the crane.



Solution

Let h represent the height (in feet) of the crane. All possible heights are greater than or equal to 4 feet *and* less than or equal to 18 feet. So, the inequality is $4 \leq h \leq 18$.



SOLVING COMPOUND INEQUALITIES A number is a solution of a compound inequality with *and* if the number is a solution of *both* inequalities. A number is a solution of a compound inequality with *or* if the number is a solution of *at least one* of the inequalities.

EXAMPLE 3 Solve a compound inequality with *and*

Solve $2 < x + 5 < 9$. Graph your solution.

Solution

Separate the compound inequality into two inequalities. Then solve each inequality separately.

$$\begin{array}{lll} 2 < x + 5 & \text{and} & x + 5 < 9 & \text{Write two inequalities.} \\ 2 - 5 < x + 5 - 5 & \text{and} & x + 5 - 5 < 9 - 5 & \text{Subtract 5 from each side.} \\ -3 < x & \text{and} & x < 4 & \text{Simplify.} \end{array}$$

The compound inequality can be written as $-3 < x < 4$.

► The solutions are all real numbers greater than -3 *and* less than 4 .



✓ GUIDED PRACTICE for Examples 2 and 3

3. INVESTING An investor buys shares of a stock and will sell them if the change c in value from the purchase price of a share is less than $-\$3.00$ or greater than $\$4.50$. Write and graph a compound inequality that describes the changes in value for which the shares will be sold.

Solve the inequality. Graph your solution.

4. $-7 < x - 5 < 4$

5. $10 \leq 2y + 4 \leq 24$

6. $-7 < -z - 1 < 3$

ANOTHER WAY In Example 3, you could solve $2 < x + 5 < 9$ by subtracting 5 from 2, $x + 5$, and 9 without first separating the compound inequality into two separate inequalities. To solve a compound inequality with *and*, you perform the same operation on each expression.

EXAMPLE 4 Solve a compound inequality with *and*

Solve $-5 \leq -x - 3 \leq 2$. Graph your solution.

$$-5 \leq -x - 3 \leq 2 \quad \text{Write original inequality.}$$

$$-5 + 3 \leq -x - 3 + 3 \leq 2 + 3 \quad \text{Add 3 to each expression.}$$

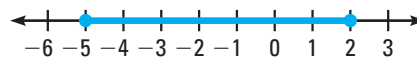
$$-2 \leq -x \leq 5 \quad \text{Simplify.}$$

$$-1(-2) \geq -1(-x) \geq -1(5) \quad \text{Multiply each expression by } -1 \text{ and reverse both inequality symbols.}$$

$$2 \geq x \geq -5 \quad \text{Simplify.}$$

$$-5 \leq x \leq 2 \quad \text{Rewrite in the form } a \leq x \leq b.$$

► The solutions are all real numbers greater than or equal to -5 and less than or equal to 2.



EXAMPLE 5 Solve a compound inequality with *or*

Solve $2x + 3 < 9$ or $3x - 6 > 12$. Graph your solution.

Solution

Solve the two inequalities separately.

$$2x + 3 < 9 \quad \text{or} \quad 3x - 6 > 12 \quad \text{Write original inequality.}$$

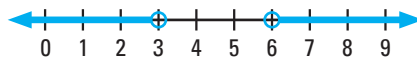
$$2x + 3 - 3 < 9 - 3 \quad \text{or} \quad 3x - 6 + 6 > 12 + 6 \quad \text{Addition or subtraction property of inequality}$$

$$2x < 6 \quad \text{or} \quad 3x > 18 \quad \text{Simplify.}$$

$$\frac{2x}{2} < \frac{6}{2} \quad \text{or} \quad \frac{3x}{3} > \frac{18}{3} \quad \text{Division property of inequality}$$

$$x < 3 \quad \text{or} \quad x > 6 \quad \text{Simplify.}$$

► The solutions are all real numbers less than 3 or greater than 6.



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GUIDED PRACTICE for Examples 4 and 5

Solve the inequality. Graph your solution.

7. $-14 < x - 8 < -1$

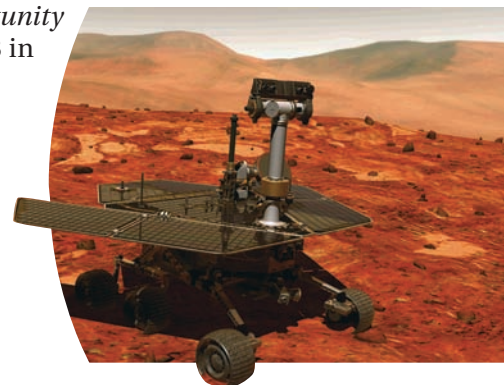
8. $-1 \leq -5t + 2 \leq 4$

9. $3h + 1 < -5$ or $2h - 5 > 7$

10. $4c + 1 \leq -3$ or $5c - 3 > 17$

EXAMPLE 6 Solve a multi-step problem

ASTRONOMY The Mars Exploration Rovers *Opportunity* and *Spirit* are robots that were sent to Mars in 2003 in order to gather geological data about the planet. The temperature at the landing sites of the robots can range from -100°C to 0°C .



- Write a compound inequality that describes the possible temperatures (in degrees Fahrenheit) at a landing site.
- Solve the inequality. Then graph your solution.
- Identify three possible temperatures (in degrees Fahrenheit) at a landing site.

Solution

Let F represent the temperature in degrees Fahrenheit, and let C represent the temperature in degrees Celsius. Use the formula $C = \frac{5}{9}(F - 32)$.

STEP 1 Write a compound inequality. Because the temperature at a landing site ranges from -100°C to 0°C , the lowest possible temperature is -100°C , and the highest possible temperature is 0°C .

$$-100 \leq C \leq 0 \quad \text{Write inequality using } C.$$

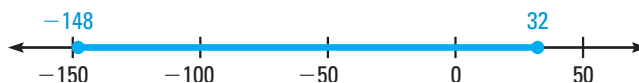
$$-100 \leq \frac{5}{9}(F - 32) \leq 0 \quad \text{Substitute } \frac{5}{9}(F - 32) \text{ for } C.$$

STEP 2 Solve the inequality. Then graph your solution.

$$-100 \leq \frac{5}{9}(F - 32) \leq 0 \quad \text{Write inequality from Step 1.}$$

$$-180 \leq F - 32 \leq 0 \quad \text{Multiply each expression by } \frac{9}{5}.$$

$$-148 \leq F \leq 32 \quad \text{Add 32 to each expression.}$$



STEP 3 Identify three possible temperatures.

The temperature at a landing site is greater than or equal to -148°F and less than or equal to 32°F . Three possible temperatures are -115°F , 15°F , and 32°F .

ANOTHER WAY

You can solve the compound inequality by multiplying through by 9:

$$-100 \leq \frac{5}{9}(F - 32) \leq 0$$

$$-900 \leq 5(F - 32) \leq 0$$

$$-900 \leq 5F - 160 \leq 0$$

$$-740 \leq 5F \leq 160$$

$$-148 \leq F \leq 32$$



GUIDED PRACTICE for Example 6

11. MARS Mars has a maximum temperature of 27°C at the equator and a minimum temperature of -133°C at the winter pole.

- Write and solve a compound inequality that describes the possible temperatures (in degrees Fahrenheit) on Mars.
- Graph your solution. Then identify three possible temperatures (in degrees Fahrenheit) on Mars.

5.4 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS**
Exs. 7, 11, and 41

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 27, 39, and 45

◆ = **MULTIPLE REPRESENTATIONS**
Ex. 43

SKILL PRACTICE

- VOCABULARY** Copy and complete: A(n) ? is an inequality that consists of two inequalities joined by *and* or *or*.
- ★ **WRITING** Describe the difference between the graphs of $-6 \leq x \leq -4$ and $x \leq -6$ or $x \geq -4$.

EXAMPLE 1

for Exs. 3–6

TRANSLATING VERBAL PHRASES Translate the verbal phrase into an inequality. Then graph the inequality.

- All real numbers that are less than 6 *and* greater than 2
- All real numbers that are less than or equal to -8 *or* greater than 12
- All real numbers that are greater than or equal to -1.5 *and* less than 9.2
- All real numbers that are greater than or equal to $-7\frac{1}{2}$ *or* less than or equal to -10

EXAMPLE 2

for Exs. 7–8

WRITING AND GRAPHING INEQUALITIES Write and graph an inequality that describes the situation.

- The minimum speed on a highway is 40 miles per hour, and the maximum speed is 60 miles per hour.
- The temperature inside a room is uncomfortable if the temperature is lower than 60°F or higher than 75°F .

EXAMPLES 3, 4, and 5

for Exs. 9–22

SOLVING COMPOUND INEQUALITIES Solve the inequality. Graph your solution.

- | | |
|---|--|
| 9. $6 < x + 5 \leq 11$ | 10. $-7 > y - 8 \geq -12$ |
| 11. ○ $-1 \leq -4m \leq 16$ | 12. $-6 < 3n + 9 < 21$ |
| 13. $-15 \leq 5(3p - 2) < 20$ | 14. $7 > \frac{2}{3}(6q + 18) \geq -9$ |
| 15. $2r + 3 < 7$ or $-r + 9 \leq 2$ | 16. $16 < -s - 6$ or $2s + 5 \geq 11$ |
| 17. $v + 13 < 8$ or $-8v < -40$ | 18. $-14 > w + 3$ or $5w - 13 > w + 7$ |
| 19. $9g - 6 > 12g + 1$ or $4 > -\frac{2}{5}g + 8$ | 20. $-2h - 7 > h + 5$ or $\frac{1}{4}(h + 8) \geq 9$ |

ERROR ANALYSIS Describe and correct the error in solving the inequality or in graphing the solution.

21.

$$\begin{aligned} 4 &< -2x + 3 < 9 \\ 4 &< -2x < 6 \\ -2 &> x > -3 \end{aligned}$$

22.

$$\begin{aligned} x - 2 &> 5 \text{ or } x + 8 < -2 \\ x &> 7 \text{ or } x < -10 \end{aligned}$$


TRANSLATING SENTENCES Write the verbal sentence as an inequality. Then solve the inequality and graph your solution.

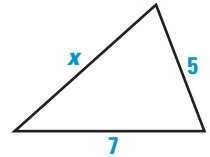
23. Five more than x is less than 8 *or* 3 less than x is greater than 5.
24. Three less than x is greater than -4 *and* less than -1 .
25. Three times the difference of x and 4 is greater than or equal to -8 *and* less than or equal to 10.
26. The sum of $-2x$ and 8 is less than or equal to -5 *or* 6 is less than $-2x$.
27. ★ **MULTIPLE CHOICE** Consider the compound inequality $a > 3x + 8$ *or* $a > -4x - 1$. For which value of a does the solution consist of numbers greater than -6 *and* less than 5?

- (A) 16 (B) 19 (C) 23 (D) 26

REASONING In Exercises 28 and 29, tell whether the statement is *true* or *false*. If it is *false*, give a counterexample.

28. If a is a solution of $x < 5$, then a is also a solution of $x < 5$ *and* $x \geq -4$.
29. If a is a solution of $x > 5$, then a is also a solution of $x > 5$ *or* $x \leq -4$.
30. Is the converse of the statement in Exercise 28 *true* or *false*? *Explain*.
31. Is the converse of the statement in Exercise 29 *true* or *false*? *Explain*.

32.  **GEOMETRY** The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- Write and solve three inequalities for the triangle shown.
 - Use the inequalities that you wrote in part (a) to write one inequality that describes all the possible values of x .
 - Give three possible lengths for the third side of the triangle.



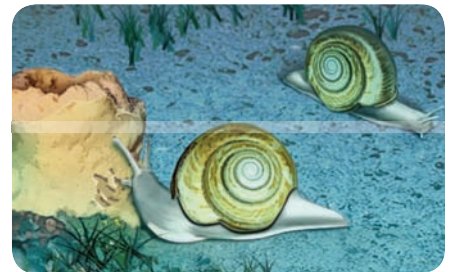
CHALLENGE Solve the inequality, if possible. Graph your solution.

33. $-18 < x - 23$ *and* $x - 16 < -22$ 34. $-3y + 7 \leq 11$ *and* $y + 4 > 11$
35. $2m - 1 \geq 5$ *or* $5m > -25$ 36. $n + 19 \geq 10$ *or* $-5n + 3 > 33$

PROBLEM SOLVING

EXAMPLE 2
for Exs. 37, 39,
40

37. **SLITSNAILS** Slitsnails are large mollusks that live in deep waters. Slitsnails have been found at elevations from -2600 feet to -100 feet. Write and graph a compound inequality that represents the elevations at which slitsnails have been found.



EXAMPLE 6
for Exs. 38,
41–43

38. **ICEBERGS** The temperature inside an iceberg off the coast of Newfoundland, Canada, ranges from -20°C to -15°C . Write and graph a compound inequality that describes the possible temperatures (in degrees Fahrenheit) of the iceberg's interior.

39. ★ **MULTIPLE CHOICE** The euro is the currency in several countries in Europe. In 2003, the dollar value of one euro ranged from \$1.0361 to \$1.2597. Which inequality represents the dollar values v that the euro was *not* worth during the year?
- (A) $1.0361 < v < 1.2597$ (B) $v < 1.0361$ or $v > 1.2597$
 (C) $1.0361 \leq v \leq 1.2597$ (D) $v \leq 1.0361$ or $v \geq 1.2597$
40. **CURRENCY** On October 25, 1865, the steamship *S.S. Republic* sank along with a cargo of gold and silver coins. The list gives the prices of several recovered gold coins. Use the least price and greatest price to write a compound inequality that describes the prices p of the coins.

Prices of Recovered Gold Coins				
\$9,098	\$20,995	\$9,798	\$33,592	\$12,597
\$16,796	\$9,798	\$10,498	\$5,319	\$73,486
\$11,897	\$32,895	\$7,349	\$6,578	\$29,395



41. (C) **ANIMALS** A deer can eat 2% to 4% of its body weight in food per day. The percent p of the deer's body weight eaten in food is given by the equation $p = \frac{f}{d}$ where f is the amount (in pounds) of food eaten and d is the weight (in pounds) of the deer. Find the possible amounts of food that a 160 pound deer can eat per day.

42. **SKIS** A ski shop sells recreational skis with lengths ranging from 150 centimeters to 220 centimeters. The shop recommends that recreational skis be 1.16 times the skier's height (in centimeters). For which heights of skiers does the shop *not* provide recreational skis?

43. ◆ **MULTIPLE REPRESENTATIONS** Water can exist as either a solid, a liquid, or a gas. The table shows the temperatures (in degrees Celsius) at which water can exist in each state.

State of water	Solid	Liquid	Gas
Temperatures (°C)	Less than 0	0 to 100	Greater than 100

- a. **Writing an Inequality** Write and solve a compound inequality to find the temperatures (in degrees Fahrenheit) at which water is *not* a liquid.
- b. **Making a Table** Make a table that gives the temperature (in degrees Celsius) when the temperature (in degrees Fahrenheit) of water is 23°F, 86°F, 140°F, 194°F, and 239°F. For which temperatures in the table is water *not* a liquid?
44. **WEATHER** Wind chill temperature describes how much colder it feels when the speed of the wind is combined with air temperature. At a wind speed of 20 miles per hour, the wind chill temperature w (in degrees Fahrenheit) can be given by the model $w = -22 + 1.3a$ where a is the air temperature (in degrees Fahrenheit). What are the possible air temperatures if the wind chill temperature ranges from -9°F to -2.5°F at a wind speed of 20 miles per hour?

45. ★ **EXTENDED RESPONSE** Some musicians use audio amplifiers so that everyone in the audience can hear the performance. The amount y of amplification per person is given by the equation $y = \frac{w}{p}$ where w is the total amount (in watts) of amplification provided by the amplifier and p is the number of people in the audience.
- Solve** Each person requires 8 watts to 10 watts of amplification. Write and solve an inequality to find the possible total amounts of amplification that an amplifier would need to provide for 300 people.
 - Decide** Will an amplifier that provides 2900 watts of amplification be strong enough for an audience of 350 people? 400 people? *Explain.*
 - Justify** Your band usually performs before an audience of 500 to 600 people. What is the least amount of amplification that your amplifier should provide? *Justify* your answer.



46. **CHALLENGE** You and three friends are planning to eat at a restaurant, and all of you agree to divide the total cost of the meals and the 15% tip equally. Each person agrees to pay at least \$10 but no more than \$20. How much can you spend altogether on meals before the tip is applied?

QUIZ

Solve the inequality, if possible. Graph your solution.

- $-\frac{1}{5}(x - 5) > x - 9$
- $\frac{1}{2}y - 8 \geq -2y + 3$
- $-4r + 7 \leq r + 10$
- $-2(s + 6) \leq -2s + 8$
- $a - 4 \geq -1$ or $3a < -24$
- $22 > -3c + 4 > 14$
- $-27 \leq 9m \leq -18$
- $5n + 2 > -18$ or $-3(n + 4) > 21$