

5.5 Solve Absolute Value Equations



Before

You solved linear equations.

Now

You will solve absolute value equations.

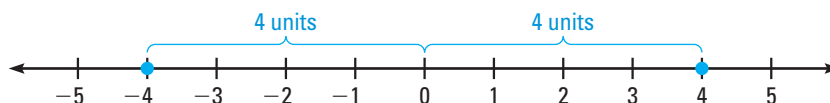
Why?

So you can analyze rules of a competition, as in Ex. 43.

Key Vocabulary

- absolute value equation
- absolute deviation
- absolute value

The absolute value of a number a , written $|a|$, is the distance between a and 0 on a number line. An **absolute value equation**, such as $|x| = 4$, is an equation that contains an absolute value expression. The equation $|x| = 4$ means that the distance between x and 0 is 4. The solutions of the equation are 4 and -4 , because they are the only numbers whose distance from 0 is 4.



COMMON CORE

CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

EXAMPLE 1 Solve an absolute value equation

Solve $|x| = 7$.

Solution

The distance between x and 0 is 7. So, $x = 7$ or $x = -7$.

► The solutions are 7 and -7 .

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GUIDED PRACTICE for Example 1

1. Solve (a) $|x| = 3$ and (b) $|x| = 15$.

SOLVING ABSOLUTE VALUE EQUATIONS In Example 1, notice that the expression inside the absolute value symbols equals 7 or the opposite of 7. This suggests the following rule for solving an absolute value equation.

KEY CONCEPT

For Your Notebook

Solving an Absolute Value Equation

The equation $|ax + b| = c$ where $c \geq 0$ is equivalent to the statement $ax + b = c$ or $ax + b = -c$.

EXAMPLE 2 Solve an absolute value equation

Solve $|x - 3| = 8$.

Solution

Rewrite the absolute value equation as two equations. Then solve each equation separately.

AVOID ERRORS

You cannot solve the equation $|x - 3| = 8$ by adding 3 to each side because $|x - 3| + 3 \neq |x|$.

$|x - 3| = 8$

Write original equation.

$x - 3 = 8$ or $x - 3 = -8$

Rewrite as two equations.

$x = 11$ or $x = -5$

Add 3 to each side.

► The solutions are 11 and -5 . Check your solutions.

CHECK $|x - 3| = 8$

$|x - 3| = 8$

Write original inequality.

$|11 - 3| \stackrel{?}{=} 8$

$|-5 - 3| \stackrel{?}{=} 8$

Substitute for x .

$|8| \stackrel{?}{=} 8$

$|-8| \stackrel{?}{=} 8$

Subtract.

$8 = 8$ ✓

$8 = 8$ ✓

Simplify. The solution checks.

REWRITING EQUATIONS To solve an absolute value equation, you may first need to rewrite the equation in the form $|ax + b| = c$.

EXAMPLE 3 Rewrite an absolute value equation

Solve $3|2x - 7| - 5 = 4$.

Solution

First, rewrite the equation in the form $|ax + b| = c$.

$3|2x - 7| - 5 = 4$

Write original equation.

$3|2x - 7| = 9$

Add 5 to each side.

$|2x - 7| = 3$

Divide each side by 3.

Next, solve the absolute value equation.

$|2x - 7| = 3$

Write absolute value equation.

$2x - 7 = 3$ or $2x - 7 = -3$

Rewrite as two equations.

$2x = 10$ or $2x = 4$

Add 7 to each side.

$x = 5$ or $x = 2$

Divide each side by 2.

► The solutions are 5 and 2.

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**GUIDED PRACTICE** for Examples 2 and 3

Solve the equation.

2. $|r - 7| = 9$

3. $2|s| + 4.1 = 18.9$

4. $4|t + 9| - 5 = 19$

NO SOLUTIONS The absolute value of a number is never negative. So, when an absolute value expression equals a negative number, there are *no solutions*.

EXAMPLE 4 Decide if an equation has no solutions

Solve $|3x + 5| + 6 = -2$, if possible.

$|3x + 5| + 6 = -2$ Write original equation.

$|3x + 5| = -8$ Subtract 6 from each side.

▶ The absolute value of a number is never negative. So, there are no solutions.

ABSOLUTE DEVIATION The **absolute deviation** of a number x from a given value is the absolute value of the difference of x and the given value:
 absolute deviation = $|x - \text{given value}|$.

EXAMPLE 5 Use absolute deviation

BASKETBALLS Before the start of a professional basketball game, a basketball must be inflated to an air pressure of 8 pounds per square inch (psi) with an absolute error of 0.5 psi. (*Absolute error* is the absolute deviation of a measured value from an accepted value.) Find the minimum and maximum acceptable air pressures for the basketball.



Solution

Let p be the air pressure (in psi) of a basketball. Write a verbal model. Then write and solve an absolute value equation.

Absolute error	=	Measured air pressure	-	Accepted air pressure	
↓		↓		↓	
0.5	=	p	-	8	

$0.5 = |p - 8|$ Write original equation.

$0.5 = p - 8$ or $-0.5 = p - 8$ Rewrite as two equations.

$8.5 = p$ or $7.5 = p$ Add 8 to each side.

▶ The minimum and maximum acceptable pressures are 7.5 psi and 8.5 psi.

GUIDED PRACTICE for Examples 4 and 5

Solve the equation, if possible.

5. $2|m - 5| + 4 = 2$

6. $-3|n + 2| - 7 = -10$

7. The absolute deviation of x from 7.6 is 5.2. What are the values of x that satisfy this requirement?

5.5 EXERCISES

HOMWORK KEY

○ = See **WORKED-OUT SOLUTIONS**
Exs. 11, 23, and 45

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 32, 44, 48, and 49

SKILL PRACTICE

- VOCABULARY** Copy and complete: The equation $|x - 7| = 0.15$ is an example of a(n) ?.
- ★ **WRITING** Given $|x - 9| = 5$, describe the relationship between x , 9, and 5 using absolute deviation.

EXAMPLES 1, 2, and 3

for Exs. 3–20

SOLVING EQUATIONS Solve the equation.

- | | | |
|------------------------------------|-------------------------|-----------------------------------|
| 3. $ x = 5$ | 4. $ y = 36$ | 5. $ v = 0.7$ |
| 6. $ w = 9.2$ | 7. $ r = \frac{1}{2}$ | 8. $ s = \frac{7}{4}$ |
| 9. $ m + 3 = 7$ | 10. $ 4n - 5 = 18$ | 11. $ 3p + 7 = 4$ |
| 12. $ q + 8 = 2$ | 13. $ 2d + 7 = 11$ | 14. $ f - 8 = 14$ |
| 15. $3 13 - 2t = 15$ | 16. $4 b - 1 - 7 = 17$ | 17. $\frac{1}{3} 2c - 5 + 3 = 7$ |
| 18. $\frac{7}{4} 3j + 5 + 1 = 15$ | 19. $4 2k + 3 - 2 = 6$ | 20. $-3 5g + 1 - 6 = -9$ |

ERROR ANALYSIS Describe and correct the error in solving the absolute value equation.

21. $|x + 4| = 13$
 $x + 4 = 13$ ✗
 $x = 9$

22. $|x - 6| = -2$
 $x - 6 = -2$ or $x - 6 = 2$ ✗
 $x = 4$ or $x = 8$

EXAMPLE 4

for Exs. 23–31

SOLVING EQUATIONS Solve the equation, if possible.

- | | | |
|----------------------------|------------------------------------|----------------------------------|
| 23. $ x - 1 + 5 = 2$ | 24. $ y - 4 + 8 = 6$ | 25. $ m + 5 + 1.5 = 2$ |
| 26. $-4 8 - 5n = 13$ | 27. $-3 1 - \frac{2}{3}v = -9$ | 28. $-5 \frac{4}{5}w + 6 = -10$ |
| 29. $-10 14 - r - 2 = -7$ | 30. $-2 \frac{1}{3}s - 5 + 3 = 8$ | 31. $-9 4p + 2 - 8 = -35$ |
32. ★ **MULTIPLE CHOICE** Which number is a solution of $|4x - 1| + 2 = 1$?
- (A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) There is no solution.

EXAMPLE 5

for Exs. 33–36

USING ABSOLUTE DEVIATION Find the values of x that satisfy the definition of absolute deviation for the given value and the given absolute deviation.

- | | |
|--|--|
| 33. Given value: 5;
absolute deviation: 8 | 34. Given value: 20;
absolute deviation: 5 |
| 35. Given value: -9.1 ;
absolute deviation: 1.6 | 36. Given value: -3.4 ;
absolute deviation: 6.7 |

37. **SOLVING AN EQUATION** Interpreted geometrically, the equation $|x - a| = b$ means that the distance between x and a on a number line is b . Solve $|x - 3| = 7$ both geometrically and algebraically. Compare your solutions.

TRANSLATING SENTENCES In Exercises 38 and 39, write the verbal sentence as an absolute value equation. Then solve the equation.

38. Four more than the absolute deviation of x from 3 is 8.
39. Five times the absolute deviation of $2x$ from -9 is 15.
40. **REASONING** Is $a|x|$ equivalent to $|ax|$ when a is positive? when a is negative? when a is 0? Give examples to support your answers.
41. **CHALLENGE** How many solutions does the equation $a|x + b| + c = d$ have if $a > 0$ and $c = d$? if $a < 0$ and $c > d$?

PROBLEM SOLVING

EXAMPLE 5
for Exs. 42–46

42. **GUARDRAILS** A safety regulation requires that the height of a guardrail be 42 inches with an absolute deviation of 3 inches. Find the minimum and maximum heights of a guardrail.
43. **CHEERLEADING** A cheerleading team is preparing a dance program for a competition. The program must last 4 minutes with an absolute deviation of 5 seconds. Find the least and greatest possible times (in seconds) that the program can last.
44. **★ MULTIPLE CHOICE** The diameter of a billiard ball must be 2.25 inches with an absolute error of 0.005 inch. What is the maximum possible diameter that a billiard ball can have?
- (A) 2.2 inches (B) 2.245 inches (C) 2.255 inches (D) 2.3 inches
45. **SPORTS** In gymnastics meets last year, the mean of your friend's least and greatest scores was 54.675 points. The absolute deviation of his least and greatest scores from the mean was 2.213 points.
- a. What were the least and greatest scores that he earned?
- b. This year the mean of his least and greatest scores is 56.738 points, and the absolute deviation of the least and greatest scores from the mean is 0.45 point. How many points more than last year's greatest score is this year's greatest score?
46. **JEWELRY** A jewelry store advertisement states that a certain diamond bracelet weighs 12 carats, but the actual weight can vary by as much as 5% of the advertised weight. Find the minimum and maximum possible weights of the bracelet.



47. **CONTESTS** You currently have 450 points in an academic contest. You choose the value p of the question you want to answer. The value p represents the absolute deviation of your new score s from 450.
- Write an absolute value equation that gives p in terms of s .
 - If you choose a question worth 150 points, what are the possible new scores that you can have after answering the question?
48. ★ **EXTENDED RESPONSE** The percent p of United States residents who were foreign born, or born outside of the United States, during the period 1910–2000 can be modeled by the equation $p = 0.165|t - 60| + 4.8$ where t is the number of years since 1910.
- Approximate** During the period 1910–2000, in approximately what year did foreign-born residents account for 13% of all residents?
 - Predict** If the model holds for years after 2000, predict the year in which foreign-born residents will again account for 13% of all residents.
 - Decide** According to the model, did foreign-born residents account for 4% of all residents at any time during the period 1910–2000? *Explain* your answer.
49. ★ **SHORT RESPONSE** A stock's average price p (in dollars) during the period February 2005 to October 2005 can be modeled by the equation $p = 2.3|m - 7| + 9.57$ where m is the number of months since February 2005.
- Approximate** In approximately what month and year was the average price \$16.15? If the model holds for months after October 2005, predict the month and year in which the average price will again be \$16.15.
 - Justify** Is it possible to use the model to estimate the stock's lowest average price during this period? *Justify* your answer.
50. **CHALLENGE** In a recent Olympics, swimmers in a men's 200 meter butterfly event finished with times from 1 minute 54.04 seconds to 1 minute 57.48 seconds. Let t represent the slowest or fastest time (in seconds). Write an absolute value equation that describes the situation.

