5.6 Solve Absolute Value Inequalities

| Before | You solved absolute value equations. |
| Now | You will solve absolute value inequalities. |
| Why | So you can analyze softball compression, as in Ex. 38. |

Key Vocabulary
- absolute value
- equivalent inequalities
- compound inequality
- absolute deviatio
- mean

Recall that $|x| = 3$ means that the distance between x and 0 is 3. The inequality $|x| < 3$ means that the distance between x and 0 is *less than* 3, and $|x| > 3$ means that the distance between x and 0 is *greater than* 3. The graphs of $|x| < 3$ and $|x| > 3$ are shown below.

Graph of $|x| < 3$
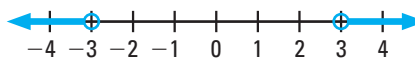
Graph of $|x| > 3$

EXAMPLE 1 Solve absolute value inequalities
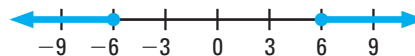
Solve the inequality. Graph your solution.

a. $|x| \geq 6$

b. $|x| \leq 0.5$

Solution

a. The distance between x and 0 is greater than or equal to 6.
So, $x \leq -6$ *or* $x \geq 6$.

▶ The solutions are all real numbers less than or equal to -6 *or* greater than or equal to 6.

b. The distance between x and 0 is less than or equal to 0.5.
So, $-0.5 \leq x \leq 0.5$.

▶ The solutions are all real numbers greater than or equal to -0.5 *and* less than or equal to 0.5.

✓ **GUIDED PRACTICE** for Example 1

Solve the inequality. Graph your solution.

1. $|x| \leq 8$

2. $|u| < 3.5$

3. $|v| > \dfrac{2}{3}$

SOLVING ABSOLUTE VALUE INEQUALITIES In Example 1, the solutions of $|x| \geq 6$ and $|x| \leq 0.5$ suggest that you can rewrite an absolute value inequality as a compound inequality.

KEY CONCEPT — For Your Notebook

Solving Absolute Value Inequalities

- The inequality $|ax + b| < c$ where $c > 0$ is equivalent to the compound inequality $-c < ax + b < c$.

- The inequality $|ax + b| > c$ where $c > 0$ is equivalent to the compound inequality $ax + b < -c$ *or* $ax + b > c$.

In the inequalities above, < can be replaced by ≤ and > can be replaced by ≥.

EXAMPLE 2 — Solve an absolute value inequality

Solve $|x - 5| \geq 7$. Graph your solution.

$$|x - 5| \geq 7$$ **Write original inequality.**
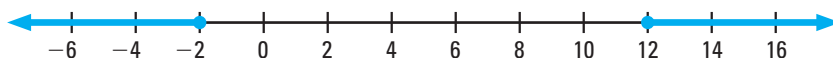
$$x - 5 \leq -7 \ \ or \ \ x - 5 \geq 7$$ **Rewrite as compound inequality.**

$$x \leq -2 \ \ or \ \ \ \ \ \ \ x \geq 12$$ **Add 5 to each side.**

▶ The solutions are all real numbers less than or equal to -2 *or* greater than or equal to 12. Check several solutions in the original inequality.

EXAMPLE 3 — Solve an absolute value inequality

Solve $|-4x - 5| + 3 < 9$. Graph your solution.

$$|-4x - 5| + 3 < 9$$ **Write original inequality.**

$$|-4x - 5| < 6$$ **Subtract 3 from each side.**

$$-6 < -4x - 5 < 6$$ **Rewrite as compound inequality.**

$$-1 < -4x < 11$$ **Add 5 to each expression.**

$$0.25 > x > -2.75$$ **Divide each expression by -4. Reverse inequality symbol.**

$$-2.75 < x < 0.25$$ **Rewrite in the form $a < x < b$.**

▶ The solutions are all real numbers greater than -2.75 *and* less than 0.25.

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✔ **GUIDED PRACTICE**   for Examples 2 and 3

Solve the inequality. Graph your solution.

4. $|x + 3| > 8$ **5.** $|2w - 1| < 11$ **6.** $3|5m - 6| - 8 \leq 13$

EXAMPLE 4 **Solve a multi-step problem**

COMPUTERS You are buying a new computer and find 10 models in a store advertisement. The prices are $890, $750, $650, $370, $660, $670, $450, $650, $725, and $825.

- Find the mean of the computer prices.

- You are willing to pay the mean price with an absolute deviation of at most $100. How many of the computer prices meet your condition?

Solution

STEP 1 **Find** the mean by dividing the sum of the prices by 10.

$$\text{Mean} = \frac{890 + 750 + 650 + 370 + 660 + 670 + 450 + 650 + 725 + 825}{10}$$

$$= \frac{6640}{10} = 664$$

STEP 2 **Write** and solve an inequality. An absolute deviation of at most $100 from the mean, $664, is given by the inequality $|x - 664| \leq 100$.

$|x - 664| \leq 100$	**Write absolute value inequality.**
$-100 \leq x - 664 \leq 100$	**Write as compound inequality.**
$564 \leq x \leq 764$	**Add 664 to each expression.**

▶ The prices you will consider must be at least $564 and at most $764. Six prices meet your condition: $750, $650, $660, $670, $650, and $725.

✓ **GUIDED PRACTICE** **for Example 4**

7. **WHAT IF?** In Example 4, suppose that you are willing to pay the mean price with an absolute deviation of at most $75. How many of the computer prices meet this condition?

CONCEPT SUMMARY *For Your Notebook*

Solving Inequalities

One-Step and Multi-Step Inequalities

- Follow the steps for solving an equation, but reverse the inequality symbol when multiplying or dividing by a negative number.

Compound Inequalities

- If necessary, rewrite the inequality as two separate inequalities. Then solve each inequality separately. Include *and* or *or* in the solution.

Absolute Value Inequalities

- If necessary, isolate the absolute value expression on one side of the inequality. Rewrite the absolute value inequality as a compound inequality. Then solve the compound inequality.

HOMEWORK
KEY

◯ = See **WORKED-OUT SOLUTIONS**
Exs. 9, 15, and 37

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 21, 22, 37, and 40

◆ = **MULTIPLE REPRESENTATIONS**
Ex. 38

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The inequalities $|x| > 8$ and $x > 8$ *or* $x < -8$ are __?__.

2. ★ **WRITING** *Describe* the difference between solving $|x| \leq 5$ and solving $|x| \geq 5$.

EXAMPLES
1, 2, and 3
for Exs. 3–24

SOLVING INEQUALITIES Solve the inequality. Graph your solution.

3. $|x| < 4$ 4. $|y| \geq 3$ 5. $|h| > 4.5$

6. $|p| < 1.3$ 7. $|t| \leq \frac{3}{5}$ 8. $|j| \geq 1\frac{3}{4}$

9. $|d + 4| \geq 3$ 10. $|b - 5| < 10$ 11. $|14 - m| > 6$

12. $|2s - 7| < 1$ 13. $|4c + 5| \geq 7$ 14. $|9 - 4n| \leq 5$

15. $5\left|\frac{1}{2}r + 3\right| > 5$ 16. $\left|\frac{4}{3}s - 7\right| - 8 > 3$ 17. $-3\left|2 - \frac{5}{4}u\right| \leq -18$

18. $2|3w + 8| - 13 < -5$ 19. $2\left|\frac{1}{4}v - 5\right| - 4 > 3$ 20. $\frac{2}{7}|4f + 6| - 2 \geq 10$

21. ★ **MULTIPLE CHOICE** Which inequality is equivalent to $x < 1$ *or* $x > 5$?

     Ⓐ $|x + 8| - 2 > 10$ Ⓑ $3|6 - 2x| > 12$

     Ⓒ $|5x + 9| < 10$ Ⓓ $|7 - 4x| - 9 < 8$

22. ★ **WRITING** How can you tell whether an absolute value inequality is equivalent to a compound inequality with *and* or to a compound inequality with *or*?

ERROR ANALYSIS *Describe* and correct the error in solving the inequality.

23.
$|x + 4| > 13$
$13 > x + 4 > -13$
$9 > x > -17$ ✗

24.
$|x - 5| < 20$
$x - 5 < 20$
$x < 25$ ✗

TRANSLATING SENTENCES Write the verbal sentence as an inequality. Then solve the inequality and graph your solution.

25. The absolute deviation of x from 6 is less than or equal to 4.

26. The absolute deviation of $2x$ from -7 is greater than or equal to 15.

27. Three more than the absolute deviation of $-4x$ from 7 is greater than 10.

28. Four times the absolute deviation of x from 9 is less than 8.

REASONING Tell whether the statement is *true* or *false*. **If it is false, give a counterexample.**

29. If a is a solution of $|x + 3| \leq 8$, then a is also a solution of $x + 3 \geq -8$.

30. If a is a solution of $|x + 3| > 8$, then a is also a solution of $x + 3 > 8$.

31. If a is a solution of $|x + 3| \geq 8$, then a is also a solution of $x + 3 \leq -8$.

32. If a is a solution of $x + 3 \leq -8$, then a is also a solution of $|x + 3| \geq 8$.

33. **CHALLENGE** Solve $|x - 3| < 4$ *and* $|x + 2| > 8$. *Describe* your steps.

34. **CHALLENGE** If $|ax + b| < c$ where $c < 0$, what is the solution of the inequality? If $|ax + b| > c$ where $c < 0$, what is the solution of the inequality? *Explain* your answers.

PROBLEM SOLVING

35. **ESSAY CONTEST** An essay contest requires that essay entries consist of 500 words with an absolute deviation of at most 30 words. What are the possible numbers of words that the essay can have?

36. **SWIMMING POOL** The saturation index for a pool measures the balance between the acid level and the amount of minerals in pool water. Balanced water has an index value of 0. Water is highly corrosive or highly scale forming if the absolute deviation of the index value from 0 is greater than 0.5. Find the index values for which pool water is highly corrosive or highly scale forming.

37. ★ **SHORT RESPONSE** You are preheating an oven to 350°F before you bake muffins. Several minutes later, the oven thermometer reads 346°F. The measured temperature has an absolute deviation of at most 2°F. Write and solve an inequality to find the possible temperatures in the oven. Should you continue to preheat the oven, or should you start baking the muffins? *Explain* your choice.

38. ◆ **MULTIPLE REPRESENTATIONS** Softball compression measures the hardness of a softball and affects the distance that the softball can travel upon contact with a bat. A softball organization requires that the compression of a softball be 350 pounds but allows an absolute deviation of at most 50 pounds.

   a. **Making a Table** Make a table that shows the absolute deviation from the required compression when the measured compression of a softball is p pounds. Use the following values for p: 275, 325, 375, 425, 475.

   b. **Writing an Inequality** Write and solve an inequality to find the softball compressions that the organization will allow. Which values of p in the table are solutions of the inequality?

○ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**   ◆ = **MULTIPLE REPRESENTATIONS**

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39. MULTI-STEP PROBLEM In a physics class, 7 groups of students experimentally determine the acceleration (in meters per second per second) of an object in free fall. The table below shows the value calculated by each group.

Group	1	2	3	4	5	6	7
Calculated value (m/sec^2)	10.50	9.52	9.73	9.86	9.78	10.90	9.86

   a. Calculate Find the mean of the measured values given in the table. Round to the nearest hundredth.

   b. Solve When writing up their lab reports, the students wanted to state that the absolute deviation of each measured value x from the mean was at most d. What is the value of d in this situation?

40. ★ EXTENDED RESPONSE *Relative absolute deviation* of a number from a given value is the absolute deviation expressed as a percent of the given value. A wildlife biologist estimates that the number of pronghorn antelope in Nevada is 18,000 with a relative absolute deviation of at most 20%.

   a. Calculate Find the absolute deviation from the estimated population of pronghorn antelope by multiplying the estimated population by the relative absolute deviation.

   b. Solve Write and solve an inequality to find the possible numbers of pronghorn antelope in Nevada.

   c. Explain If the relative absolute deviation were 25%, could you conclude that the actual population is necessarily greater than if the relative absolute deviation were 20%? *Explain* your reasoning.

41. CHALLENGE According to the rules for a women's figure skating event, a skater should finish a routine in an ideal time of 3 minutes 30 seconds. The skater receives a 0.1 point penalty if the absolute deviation of the finishing time from the ideal time is greater than 10 seconds *and* less than or equal to 20 seconds. Write and solve an inequality to find the finishing times for which the skater receives a 0.1 penalty point.